A COHESION MODEL OF MICROCRACK TOUGHENING

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Abstract—A cohesion model of microcrack toughening is presented. This model is based on the existence of a $K$-field around the microcrack process zone. This $K$-field and the untracked material toughness dictate the unstable crack propagation in brittle materials. The toughening mechanism and the $R$-curve effect are well described by the model. The toughening magnitude of the microcrack process zone can be easily predicted by the size and the cohesive strength of the microcrack process zone. A comparison between theory and experiment is made and reveals good agreement.

1. INTRODUCTION

The incidence of microcracking to form a process zone around main crack tip is now a well-substantiated phenomenon for brittle solids such as ceramics and rocks [1-4] and there is growing evidence [5-8] that the process zone may postpone the outset of unstable macroscopic crack propagation and is a viable toughening mechanism. The present theoretical analyses [8-10] establish considerable similarities between transformation and microcrack toughening, by relating the change in toughness to the permanent dilatation and the reduced modulus in the process zone. This approach provides some insight into the toughening mechanism of the microcrack process zone. As will be elaborated in the following section, however, due to some theoretical deficiencies and profuse material parameters required by the theoretical model, which are very difficult to determine by experiment, it is very difficult to test quantitatively the validity of the present mechanical model. There are various topics required to be further studied before authoritative conclusions can be reached regarding microcrack toughening and before an explicit prediction of toughening trend can be contemplated. The intent of this study is to establish a simple mechanical model, as is necessary to describe the microcrack toughening mechanism and the $R$-curve phenomenon, and to make an explicit prediction of the toughening trend from relatively little experimental data, which can be easily measured.

2. DISCUSSION ON MICROCRACK TOUGHENING

The present theoretical analyses of the microcrack toughening focus attention on predicting the stress intensity change $\Delta K$, defined as

$$\Delta K = K_{\text{tip}} - K_{\infty},$$

where $K_{\text{tip}}$ is the stress intensity in the microcracking process zone immediately ahead of the crack tip; $K_{\infty}$ is the stress intensity determined by the applied loads. When $K_{\text{tip}} < K_{\infty}$, $\Delta K$ is negative and the process zone shields the tip from the applied loads. The near tip field provides the crack extension criterion

$$K_{\text{tip}} = 3.3 (2)$$

where $K_{\text{tip}}$ is the fracture resistance of the microcrack degraded material. The observed toughness is

$$K_{\text{ic}} = K_{\infty} + \Delta K_{\text{tip}}$$

where $\Delta K_{\text{tip}}$ is the quantity $-\Delta K_{\text{tip}}$, evaluated at the fracture criticality governed by $K_{\text{tip}}$ and $K_{\infty}$.

There are two sources of changes in $K_{\text{tip}}$. First, microcracks invariably form in regions of residual tensile stresses, caused by thermal expansion mismatch [8, 9], transformation [11, 12], etc.
Relief of this residual stress by microcracking results in residual opening, and associated dilatation. The dilatation is the prime source of microcrack toughening, analogous to the role of dilatation in transformation toughening. A continuum mechanics model originally developed by Budiansky et al. has been used to describe the steady state toughness increases in transformation-toughened materials [13] and later extended to microcracking materials [9]. This model treats the process zone as a continuum of dilatant particles near a crack tip. The increase in toughness due to dilatation, $\Delta K_d$, depends on the process zone size and shape. For a steady state supercritical condition [10, 12]

$$\Delta K_d' = - \lambda E f \theta^T h^{1/2},$$

where $E$ is the Young modulus and $\lambda$ is a coefficient, dependent on the selected microcrack initiation criterion; $h$ is the width of the process zone (Fig. 1); $f$ is the volume fraction of transforming particles; and $\theta^T$ is the volume strain, as governed by the microcrack shape and the prior residual tensile stress.

Secondly, the process zone results in, on average, modulus reduction, which may shield the crack tip from the applied stress field. The present approximations of the elastic shielding effect replace the entire microcracking process zone by a continuum medium of some appropriate effective modulus $E$ and this continuum medium is assumed to extend down to the very microscopic crack tip [10]. The steady-state shielding induced by modulus reduction, $\Delta K_m^\mu$, is independent of the size of the process zone [9, 10], but has some dependence on zone shape and the form of the constitutive law that governs microcracking. A close approximation for $\Delta K_m^\mu$ is [10]

$$\Delta K_m^\mu = K_m \left[ 1 - (k_1 - \frac{1}{3}) \delta_1 - (k_2 + \frac{1}{3} \delta_2) \right]^{-1}$$

$$\delta_1 = \frac{1}{1 - v} \left( \frac{G}{G - 1} \right); \quad \delta_2 = \frac{1}{1 - v} \left( \frac{\bar{\nu} G}{G - \bar{\nu}} \right).$$

where $k_1 \approx 0.017$, and $k_2 \approx -0.043$, as appropriate for microcracks governed by a critical mean stress, $G$ and $\nu$ are the shear modulus and the Poisson ratio, and the overbar refers to microcracked material.

Considering the fact that at the crack tip the modulus of the material must be that of the virgin-matrix material, not that of the reduced modulus, Hutchinson proposed a modified continuum model which contains an inner near-tip spherical zone of arbitrarily small size with the original matrix modulus [10]. Then the effect of the modulus reduction shielding is notably decreased [14, 15].

The contributions $\Delta K_d'$ and $\Delta K_m^\mu$ are assumed to be additive, i.e.

$$\Delta K'_i = \Delta K_d' + \Delta K_m^\mu.$$

A number of substantive problems existing in the theoretical prediction of the toughening increment $\Delta K_i$ are as follows.

(i) Limited knowledge of interactions between modulus and dilatational contributions to the microcrack shielding as well as between microcracks or microcracks and the main crack.

(ii) The microcrack density will tend to decrease with distance from the main crack tip, resulting in a diffuse process zone. The microcrack density near the main crack tip may approach

Fig. 1. Crack growth resistance curve.
unity. It is in this region that the condition for crack extension may be determined by the linking of the microcracks to the main crack tip. The present methods, however, are not fundamentally satisfying.

(iii) The observed size of the process zone in ceramics [16, 17] is essentially in the scale of microstructure, the microcrack length in which is less than the size of a grain facet in single-phase material or of the particle phase in two-phase or multiphase material. Therefore, other local factors, such as grain orientation, anisotropy of the fracture energy on different crystal planes, etc. also have important influence on the fracture characteristic parameters $K_{up}$ and $K_0$. For example, stochastic microcrack arrays near the main crack tip may lead to a large fluctuation on the $K_{up}$-field [14, 15] and produce a noticeable mode II stress intensity factor under mode I remote loading due to microcrack asymmetries [15]. The $K$-field can be considered to dominate the local stress and strain over a region which must be much larger compared with the scale of microstructural deformation. This implies that it would be suspected whether the $K_{up}$-field exists in the process zone.

On top of the aforementioned theoretical deficiencies, the calculation of $K_{up}$ relates to such quantities as the number density, the length distribution and the residual opening of the microcracks. Theoretical prediction or measurement for these quantities is very difficult. For example, the dilatation induced by the residual opening can be estimated provided that prior residual tensile stress on the microcrack plane is known. It is in itself a very hard problem to visualize the distributions of the residual stress system developed in the fabrication of polycrystalline or multiphase materials. The spatial variation of the residual stress is set by the grain size or by the scale of second phase particles. Moreover, the present calculation for dilatation involves only the residual opening caused by the relief of the residual tensile stress. No attention has been paid to the estimation of the closure effect caused by the relief of the residual compressive stress, which does exist near the microcrack.

Therefore pertinent estimation of the dilatation is also a hard problem.

A more substantive problem is a poor fundamental understanding of the degradation of the materials in the process zone due to microcracks. On the other hand, three-dimensional locations of the microcracks would notably reduce the constraint strength of the main crack tip. The stress triaxiality is one of the important parameters with which to characterize the full range of near tip fracture environments [18–21]. In point of the reduction in stress triaxiality caused by microcracks, the toughness of the process zone should be enhanced. It can be imagined that theoretical analysis or experimental measurement of the toughness of the microcrack degraded material is very difficult.

An important characteristic of the microcrack toughening is the existence of resistance ($R$) curve behaviour [3, 4, 8, 22, 23]. When the near tip field $K_{up}$ reaches the critical value $K_0$ the crack growth starts. The crack growth progresses quasi-statically if the wake of the microcracked material left behind the advancing tip would reduce the intensity of $K_{up}$. Consequently, the applied intensity $K_\infty$ must increase with crack advance $\Delta a$ to produce a resistance curve such as that shown in Fig. 1. For a wake of uniform width, $h$, the $R$-curve should exhibit an asymptote [9, 24] when the crack advance $\Delta a \geq 2 \sim 3h$. A continuously increasing $R$-curve is only expected when the zone wake evolves during crack extension. From Fig. 1 it can be seen that because $K_0$ is difficult to evaluate one cannot make an explicit determination from the evaluated $\Delta K'$ as to whether the value of $K_0 + \Delta K'$ is lower or greater than the uncracked matrix toughness $K_{up}'$, i.e. one cannot identify from the calculated $\Delta K'$ whether or not the microcrack process zone provides a viable toughening. Since the value of $\Delta K'$ increases with increasing dilatation and modulus reduction (i.e. microcrack density, length, etc.), while the degraded toughness $K_0$ decreases with these values, theoretical analyses should seek for an optimal combination between $\Delta K$ and $K_0$. To do this one must establish simultaneously $K_0$ as a function of the dilatation and modulus reduction. As discussed above this is a very difficult problem. In addition, the above theory cannot explain a large quasi-static crack growth ($\Delta a \geq 3h$) as observed in the actual testing [17, 25].

3. THE COHESION MODEL

As mentioned above, there are various substantive problems on the microcrack toughening theory based on the field analysis in the process zone. Since the size of the microcrack process zone
is very small and the $K$-field around the process zone exists (Fig. 2a), unstable crack extension should be dictated by the criterion of

$$K \geq K_{\text{tip}},$$

rather than $K_{\text{tip}} \geq K_0$, where

$$K = K_0^* + \Delta K^*$$

takes account of the shielding effect of the microcrack process zone on the applied stress intensity $K_0^*$, which is indicated by the value of $\Delta K^*$. It is expedient to estimate $K$ in terms of a cohesion model as shown in Fig. 2b. In this model, the main crack tip is set at the profile of the process zone and the role of the process zone is replaced by a cohesive stress. Consequently, the $K_0^*$ is evaluated by a modified crack length $a + \rho$ and is slightly different from $K_0$ calculated with the crack length $a$. Because the size of the cohesive zone is very small compared with other dimensions, the shielding effect produced by the cohesive stress, $\Delta K^*$, can be evaluated by the formula of the semi-infinite crack subjected to surface traction [26], i.e.

$$\Delta K^* = \frac{2}{\sqrt{(2\pi)}} \int_0^\rho q Y^{-1/2} \, dY,$$

where $Y$ is the distance from the crack tip and $q$ is the distributive intensity of the bond force. If the shielding effect $\Delta K^*$ before unstable propagation is to be determined, it is necessary to construct an appropriate constitutive equation for the damage material as a function of microcrack density parameter and $K_0^*$, etc. to determine $q$, and the analysis is similar to that discussed in ref. [27]. However, at the time at which the crack begins to unstably propagate, the bond force mainly depends on the microcrack density and the material strength. A crude estimate of the cohesive strength is as follows:

$$q = \sigma_f (1 - \alpha),$$

where $\sigma_f$ is the fracture strength of the material and $\alpha = N(l)$ is a crack density parameter, as described by Budiansky and O’Connell [28], where $l$ is the microcrack length and $N$ the number density per unit volume. Equation (11) gives an upper limit estimate of the cohesive stress since the effect of stress concentration induced by the microcrack on the reduction of the cohesive strength is not taken into account.

The applied stress intensity $K_0^*$ is

$$K_0^* = F o_0 \sqrt{(a + \rho)},$$

where $F$ is the geometry factor of the cracked sample, and $o_0$ is the applied stress. Due to $\rho \ll a$, we have

$$K_0^* \approx K_0 + \Delta K_0,$$

Fig. 2. Schematics illustrating the microcrack process zone (a) and associated cohesion model (b).
The net shielding effect of the process zone is
\[ \Delta K = \Delta K^* + \Delta K_\infty. \]

For a uniform cohesive stress, eq. (10) can be written as
\[ \Delta K^* = \frac{4}{\sqrt{(2\pi)^3}} q \sqrt{a \left( \frac{\rho}{a} \right)^{1/3}}. \]

Because the cohesive stress \( q \) is, in general, much larger than the applied stress, \( \sigma_\infty \) and \( \rho/a \approx 1 \), the absolute value of \( \Delta K^* \) is much larger than that of \( \Delta K_\infty \), i.e. the existence of the process zone has an evident shielding effect.

The microcrack toughening mechanism and \( R \)-curve characteristic can be described by the model as schematically shown in Fig. 3. In Fig. 3, the \( K^*_\infty \)-curve shows the variation of the applied stress intensity with the applied stress. The values corresponding to the points \( A_1 \) and \( A_2 \) on the \( K^*_\infty \)-curve, \( K_f \) and \( K_\theta \), indicate the initiations of the microcracking and main crack growth, respectively. It is evident that \( K_f \) must be lower than the uncracked matrix toughness in order to form a microcrack process zone prior to unstable crack extension. The actual value of the \( K_f \) is dependent on the residual stress field and the microstructure of the matrix material. The value of \( K_\theta \) is, in general, lower than that of the matrix due to the fact that the toughness of the microcrack degraded material is lower than that of the matrix. After \( K^*_\infty \) exceeds \( K_f \), the microcrack process zone evolves and the size of the process zone, \( \rho \), increases with increasing \( K^*_\infty \) as shown by the \( \rho \)-curve. At the same time, the shielding effect of the process zone enhances with increasing size of the process zone, which leads to a gradually increasing reduction in the stress intensity \( K \) from the applied stress intensity \( K^*_\infty \) as shown by the \( K \)-curve. When \( K^*_\infty \) increases to \( K_f \), at which a stable value of the size of the process zone and a sufficient wake with width, \( h \), has been formed, the shielding effect of the process zone reaches a maximum value of \( \Delta K_f \). Exceeding \( K_f \), \( K^*_\infty \) and \( K \)
increase in parallel with increasing applied stress. When $K$ reaches the matrix toughness, the unstable crack growth occurs and the corresponding value on the $K^{*}$-curve is the observed toughness.

The characteristic of the $R$-curve is described by the dashed line in Fig. 3. When the applied stress intensity $K^{*}$ reaches $K_{0}$, the main crack begins to advance and the $R$-curve has great slope due to increase in the shielding effect of the process zone. During quasi-statically crack growth, part of the external work is dissipated by main crack extension and microcrack process zone evolution, and the rest is used to increase the elastic deformation energy of the cracked body. When the applied $K^{*}$ exceeds $K_{c}$, the energy dissipated by main crack growth and the microcrack process zone evolution reaches a saturate value, and the enhancement of the crack growth resistance is due to the fact that before the crack grows unstably the elastic deformation energy stored in the cracked body must be increased to a critical value, at which the stress intensity $K$ ahead of the zone profile reaches $K_{c}^{m}$. Therefore, this model is able to explain large quasi-static crack growth as observed in a number of actual tests [3, 4, 17, 25, 29], while the continuum models allow a relatively small amount of advance amounting to only two or three times the half-height of the zone.

Unstable crack growth is controlled by the $K$-field, which must be able to describe the stress environment ahead of the process zone. The stress distribution described by the cohesion model is schematically plotted in Fig. 4. The solid curve indicates the stress field modified by the process zone from the initial linear elastic field. Because the microcrack density decreases with distance from the crack tip and the plane strain condition at the zone profile is transformed to be close to a plane stress condition near crack tip, the cohesive stress decreases with distance from crack tip. In order to balance the stress released by the zone, the stress ahead of the zone profile (DEF line) must be higher than that described by the $K_{c}$-field (BGH line), which makes the area of BEF equal to that of ABC. It is also evident that the stress intensity ahead of the zone is lower than that of the $K^{*}$-field with the crack tip at the zone profile. A pertinent representation of the stress environment would be given by the $K$-field as shown in Fig. 4. A direct simulation of creation of the microcrack process zone during loading (Fig. 5) was conducted by finite element calculation on a single edge crack plate under a plane stress condition (the detail will be given elsewhere). The microcrack array and density were random and uniform. The microcrack length varied in the range of $3.1-8.0 \times 10^{-4} \, l/a$. The cohesive node forces in the process zone can be precisely evaluated, and then the stress intensity $K$ can be calculated from eqs (10), (12) and (13). As shown in Fig. 5, the stress intensity near the process zone is well described by the $K$-field. It is noted that the stress distribution in the process zone shows great fluctuation, which depends on the local microcrack configuration and cannot be fitted by a $K_{m}$-field.

![Fig. 4. Schematic comparison of the stress intensity around the microcrack process zone with that determined by $K_{m}$, $K^{*}$ and $K$-fields.](image-url)
4. COMPARISON WITH EXPERIMENT

More observational data such as the number density, length distribution and residual opening of the microcracks in the process zone are needed before it will be possible to settle on any continuum model. In addition, a deeper understanding is called for of the relation between the inherent fracture toughness of the microcracked material and these microcrack parameters. Because of the experimental difficulty involved in detecting microcracks (typical residual crack openings are less than 2 nm) there is very little definitive experimental data on the toughening increment due to microcracking. A validated example of this type has been made for zirconia toughened alumina (ZTA) by Rühle et al. [12, 16, 17], who processed ZTA ceramics with a fixed total content of 15 vol.% ZrO₂.

The processing parameters varied such that the volume fraction $f$, of tetragonal ZrO₂($t$-ZrO₂) varied from 0.23 and 0.86. The measured fracture toughness and flexural strength are represented in Fig. 6. All of the material had appreciably higher toughness than the pure matrix $\mathrm{Al₂O₃}(K_C \approx 3.5 \text{ MPa } \sqrt{\text{m}})$. Until now, the different microcrack parameters, which are necessary for the theoretical analysis of the continuum model, have been determined for two materials, one primarily containing monoclinic ZrO₂($m$-ZrO₂)($f_1=0.23$, $\sigma_1 \approx 740 \text{ MPa}$, $K_{IC} \approx 6.2 \text{ MPa } \sqrt{\text{m}}$) and the other containing about equal volume fractions of $t$-ZrO₂ and $m$-ZrO₂($f_2=0.46$, $\sigma_2 = 950 \text{ MPa}$, $K_{IC} = 5.9 \text{ MPa } \sqrt{\text{m}}$). For the analysis based on the cohesive model, the necessary microcrack parameters are only the microcrack density distribution and the size of the microcrack process zone, which are summarized in Fig. 7 for the two materials. The microcrack density diminishes with distance from the crack plane. The decrease with distance $y$ is approximately linear, such that

$$\alpha = \alpha_0 (1 - y/h),$$

where $h$ is the process zone width, $\alpha_0$ is maximum density. The data pertinent to material I (23 vol.% $t$-ZrO₂) are $\alpha_0 = 0.165$, $h = 6.9 \mu\text{m}$ and to material II (4.6 vol.% $t$-ZrO₂) are $\alpha_0 = 0.21$, $h = 3.6 \mu\text{m}$. Substituting these values of $\alpha_0$ and $h$ and corresponding flexural strength into eqs (11) and (10) results in toughening increments due to microcracking listed in Table 1 for the two materials. To compare, the measured $\Delta K_m$ ($-K_{IC} - K_{IC}^R$) and the value predicted by the continuum model are also listed in Table 1. It can be seen from Table 1 that the toughening increments calculated by the cohesion model agree well with the measured one, while that calculated by the continuum model is considerably larger than the measured value. It should be noted that the measured $\Delta K_m$ includes two parts, one contribution caused by microcracking, the other due to
t-ZrO₂ → m-ZrO₂ transformation. The toughening increment induced by t → m transformation for the material I predicted by the continuum model [17] is 1.2 MPa \( \sqrt{m} \). It is assumed that there is a simple additivity of transformation and microcrack toughening, i.e. their interactions are neglected. Because the observed microcracks primarily occur around the m-ZrO₂ grains and have the features schematically shown in Fig. 7, it is reasonable to expect that the radial microcracks around the m-ZrO₂ particle would partially relieve the compressive stress induced by transformation dilatation. Consequently, the transformation toughening would be lowered by the accompanying microcracking.

5. CONCLUDING REMARKS

In view of the existence of the K-field around a microcrack process zone, unstable crack growth should be controlled by the K-field and the matrix toughness.

The features of the K-field, the toughening mechanism and R-curve phenomenon of the microcrack process zone are well characterized by the cohesion model. The maximum toughening increment induced by the microcrack process zone at unstable crack growth may be predicted by the size and the remaining cohesive strength of the damaged material of the process zone. This model does not involve the reason for the crack initiation and quasi-static growth, which are dependent on the microscopic behaviour of the process zone. There are various nebulous problems which require further study before an explicit representation of this procedure is given.

REFERENCES

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