Interfacial fracture mechanics on sandwich specimens with variable layer thickness *

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In this paper the influence of an elastic interlayer on the mechanical performance of an otherwise brittle material is analyzed in terms of interface stress intensity factors and crack tip loading mode. Dundurs' parameters are applied in order to characterize the elastic heterogeneity of the systems under investigation. It is found that critical interlayer thicknesses exist below which the second interface interacts with the near tip stress field of the interface crack. This critical thickness is different for thermal loading, $\frac{d_{\text{res}}}{w} = 1.25$, and mechanical loading, $\frac{d_{\text{app}}}{w} = 0.75$, where $w$ is the width of the specimen. Crack tip stress intensities as well as mode mixities as a function of elastic mismatch and layer thickness deviate stronger from the respective bimaterial cases under thermal loading compared to mechanical loading. Stress intensity peak values are observed around $\frac{d}{w} = 0.5$ with respect to thermal loading.

1. Introduction

Layered composites frequently fail at or in the vicinity of interfaces after fabrication or during service loading, e.g. [1]. Interfacial failure analysis has thus attracted much attention and many studies have been performed in order to gain better understanding of the underlying mechanics, e.g. [2–7]. The influence of the thickness of an elastic interlayer in an otherwise homogeneous body has been long known to influence the mechanical behaviour of layered composites. For instance, it is well known that “thin” interlayers have a negligible residual thermal stress effect on the failure characteristics in applications such as brazed metal/ceramic components. Moreover, thermal residual stress induced fracture is observed in sandwiched glass materials (fig. 1) which would never fail when bonded as a bimaterial [8]. On the other hand, bimaterials are prone to a variety of interfacial failure characteristics due to combined thermal residual or externally applied stresses [9,10]. However, in literature a detailed analysis of the effects of elastic interlayer thickness on interfacial failure is lacking so far.

2. Problem formulation

Interfacial failure under combined thermal and external loading has been the topic of recent studies in this field [4–6,8–10]. It has been pointed out that the effective stress intensity factor (SIF) at interface crack tips can be written as a superposition of thermal residual (res) and mechanically applied (app) loading

$$K_{\text{eff}} = K_{\text{app}} + K_{\text{res}}.$$  

(1)

This effective SIF is complex in nature and thus contains mode I and mode II contributions as for the bimaterial case. The SIFs on either side of eq. (1) possess the general form

$$K = YT\sqrt{a} a^{-\imath \epsilon} e^{\imath \phi},$$  

(2)
Fig. 1. Crack appearance in an SF53/SF56/SF53 sandwich of glasses after cooling to room temperature ($d/w = 0.229$).

Fig. 2. Situations considered: (a) thermal and (b) four-point bending loading.
where \( a \) is the crack length (fig. 2), \( T \) represents the nominal loading of the system

\[
T = \begin{cases} 
E^* \Delta \alpha \Delta T & \text{(thermal loading)} \\
(3FL)/(2tw^2) & \text{(mechanical loading)}
\end{cases}
\]

and the composite Young's modulus

\[
\frac{1}{E^*} = \frac{1}{2} \left( \frac{1}{E_1^*} + \frac{1}{E_2^*} \right)
\]

is given in terms of Young's moduli of the components \( i = 1, 2 \)

\[
E_i^* = \begin{cases} 
E_i/(1 - \nu_i^2) & \text{(plane strain)} \\
E_i & \text{(plane stress)}
\end{cases}
\]

\( \Delta \alpha = \alpha_1 - \alpha_2 \) is the difference in thermal expansion coefficients and \( \Delta T \) is the difference between processing and service temperature. The correction function, \( Y(\alpha, \beta, \alpha/w, d/w) \), is depending on the crack length and the layer thickness \( d \) as well as on the elastic mismatch represented by the two Dundurs' parameters [11]

\[
\alpha = \frac{k(\kappa_1 + 1) - (\kappa_2 + 1)}{k(\kappa_1 + 1) + (\kappa_2 + 1)}, \quad (6a)
\]

\[
\beta = \frac{k(\kappa_1 - 1) - (\kappa_2 - 1)}{k(\kappa_1 + 1) + (\kappa_2 + 1)}, \quad (6b)
\]

where \( k = \mu_2/\mu_1 \) is the ratio of shear moduli of both materials, \( \kappa_i = (3 - 4\nu_i) \) for plane strain and \( \kappa_i = (3 - \nu_i)/(1 + \nu_i) \) for plane stress is Muskelishvili's constant [12], and the index \( i = 1, 2 \) refers to material "1" and "2", respectively. Schmauder [13] has shown that the expression for \( \alpha \), eq. (6a), can be simplified in terms of the Young's moduli of the phases,

\[
\alpha = \frac{E_2^+ - E_1^+}{E_2^+ + E_1^+}.
\]

The bimaterial constant of the system is merely a function of \( \beta \),

\[
\epsilon = \frac{1}{2\pi} \ln \left( \frac{1 + \beta}{1 - \beta} \right)
\]

and \( \psi \) characterizes the phase angle of the SIF \( K_\alpha \) at the crack tip.

The finite element method (FEM) was applied to calculate the interracial crack tip correction functions and phase angles of loading for thermal and externally applied loading according to the boundary conditions shown in fig. 2. A typical finite element mesh of the problem is shown in fig. 3 consisting of 8-noded rectangular elements and the FE-program PERMAS was used [14]. Previously, the method has been shown to provide consistent and reliable results [8,15].

![Fig. 3. Typical finite element representation for the sandwich geometry with \( a/w = 0.5 \) and \( d/w = 0.76 \).](image-url)
3. Results

In the following the influences of thermal residual stresses, external applied stresses, layer thickness ratio, elastic mismatch and materials stacking sequence on the complex SIF is investigated. The crack length was kept constant at \( a/w = 0.5 \) within this study. In this work we restrict ourselves to the case of \( \beta = 0 \) as the influence of \( \beta \) on \( Y \) and \( \psi \) was shown to be negligibly small in a previous study on bimaterials [6]. \( \alpha \) is varied within the limits \(-0.6 \leq \alpha \leq +0.6\), as many material combinations follow this restriction [16]. Comparisons will be made for correction functions of sandwiches, \( Y_{\text{sandw}} \), with the respective bimaterials, \( Y_{\text{bi}} \), for both loading cases.

3.1. Thermal residual stresses

Figure 4 depicts several features of the thermal residual stress loading case of sandwiched materials with an interface crack of length \( a/w = 0.5 \). First, it is noted that no matter whether the interfacial layer is stiffer or softer than its surroundings the thermal residual SIF is elevated up to 20\% compared to the bimaterial case for thicknesses, \( d/w \leq 1.5 \) and the maximum is located at \( d/w \approx 0.5 \) for the regime of \( \alpha \)-values examined. This elevation is reduced to zero for vanishing interlayer thicknesses, \( d/w \to 0 \), as expected.

For thermal loading, several interesting features can be derived: first, soft interlayers (\( \alpha > 0 \)) should be used in order to achieve low SIF’s if a certain thickness of the interlayer cannot be avoided. Second, if the interlayer thickness is prescribed to be \( d/w > 0.5 \) then a stiff interlayer (\( \alpha < 0 \)) should preferred. Third, the worst case for the analyzed geometry is obtained in case of a strip of material with thickness \( d/w = 0.5 \) with a different thermal expansion coefficient within an otherwise homogeneous material.

Accordingly, the mode mixity for sandwiched materials, \( \psi_{\text{sandw}} \), differs from that of bimaterials, \( \psi_{\text{bi}} \), for layer thicknesses \( d/w \leq 1.5 \) (fig. 5). However, the mode I contributions which evolve compared to the dominant mode II contributions in the case of bimaterials [6] remain moderate below 25\° for \( 0.2 \leq d/w \leq 1.5 \). Soft elastic interlayers experience higher mode I contributions compared to the case of stiffer interlayers.

It has to be kept in mind that interlayers with higher thermal expansion coefficients compared to the surrounding material (\( \Delta \alpha = \alpha_1 - \alpha_2 > 0 \)) do have stabilizing effects in terms of interfacial SIFs while interlayers with lower thermal expan-
sion coefficient open the crack tip and, therefore, provide additional crack driving forces. Recent stress optical measurements seem to confirm these findings [8].

3.2. Applied mechanical stresses

In contrast to the previous loading case, the influence of interlayer thickness in case of external loading on applied SIF as well as mode mixity at interfacial cracks is less pronounced and limited to thin interlayers of thickness \( d/w < 0.75 \) (figs. 6 and 7).

Compared to the respective bimaterials, the applied SIFs of interface cracks increase in case of stiffer interlayers and vice versa (fig. 6) confirming earlier findings of reduced applied SIFs when an interlayer is present, such as in the case of a metal between ceramics [2]. However, stiffer interlayers result in slightly less mode II contributions than soft elastic interlayers, although these mode II contributions are very limited. The differences with respect to SIFs and mode mixity variations between interlayers and bimaterials are much smaller for applied forces compared to the case of residual stresses.

4. Concluding remarks

It has been shown that elastic interlayers of thickness \( d/w > 1.5 \) behave like an adjoint bimaterial for any type of loading. Residual thermal stresses, however, evolve to influence an interfacial crack for layer thicknesses of \( d/w < 1.5 \). Their contribution to the effective SIF at the crack tip is maximum for \( d/w = 0.5 \) for the range of bimaterials studied and was found to be in the order of up to 20% of the residual stress intensities for the respective bimaterials. Applied stresses in four-point bending contribute to the effective stress intensities for interlayer thicknesses below \( d/w < 0.5 \) providing an increase in effective SIFs for stiffer interlayers, only. Soft elastic interlayers provide reduced applied stress intensities. In contrast, thermal residual SIFs reduce to zero when the interlayer thickness is decreased.

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