FINITE ELEMENT FRACTURE ANALYSIS OF WC-CO ALLOYS

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Abstract—An elastic-plastic finite element analysis of the crack-tip field in a WC-Co alloy was performed
to achieve a detailed understanding of ductile fracture in the Co-phase. A model in which a Co-phase
was embedded at the crack-tip in an elastic solid was employed, and Gurson's constitutive equations for
a porous plastic material were used for the Co-phase in order to take into account the nucleation and
growth of microvoids. The effects of the shape of Co-phase and the stress state (plane stress or plane strain)
on the distributions of hoop stress, hydrostatic stress and microvoid volume fraction were discussed based
on the computational results. The process of ductile fracture under constraint of deformation is also
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INTRODUCTION

The WC-Co hard alloys (hereafter these will be abbreviated as WC-Co) have high hardness
as well as high wear resistance, and hence are widely used or will be widely used for
machining tools and structural parts of various machines. To improve the strength of the WC-Co,
it is necessary to understand the fracture behaviors of these alloys, which consist of WC crystals
and Co binder (hereafter these will be referred to as WC and Co, respectively), as shown in
Fig. 1 [1].

Togo et al. [2, 3] performed a finite element analysis of the stress field near a crack tip. The
results show that micro cracks are at first nucleated by fracture of WC or debonding between WC
and Co, and then ductile fracture of Co occurs followed by crack extension. It is suggested from
these results that Co plays an important role for improving the fracture toughness of WC-Co. Sato
and Honda [4] have shown by conducting an experiment that the fracture toughness of WC-Co
increases with an increase in the volume fraction of Co. Sigl and co-workers [5, 6] measured the
actual shape of WC and Co, and performed a finite element calculation for the measured shape
of WC and Co to discuss the fracture of Co.

In this study, a finite element analysis is performed to examine the effect of the shape of Co
near a crack tip on its fracture behaviors. The Co, which is much softer than the WC, is subjected
to the deformation constraint from the surrounding WC crystals. The study of the ductile fracture
under these constraints may be important not only in WC-Co, but also for various composite
materials.

NUMERICAL PROCEDURES

Let us consider a WC-Co plate with a crack and model (Figs 2 and 3), where a rect-
angular or rhombic Co-region is located at the tip of a sharp notch. We assume that the
small scale yielding condition is satisfied and the solid surrounding the Co in Figs 2 and 3 is
an elastic body with the average material constants of the WC-Co composite (WC:90wt% +
Co:10wt%) as shown in Table 1. We consider three different shapes of Co as shown in
Fig. 3.
We also assume that Co is an elastic–plastic body and employ Gurson’s constitutive equation [7] to take account of the effect of nucleation and growth of microvoids. The yield function is given by

\[ \Phi = \frac{3}{2} \frac{\sigma_y \sigma_y'}{\sigma_m^2} + 2f \cosh(\Sigma) - [1 + f^2] = 0 \]  

where \( \sigma_y \) is the macroscopic true stress, \( \sigma_y' = \sigma_y - \delta \sigma_{kk}/3 \) is the stress deviator, \( \delta \) is Kronecker’s delta, \( \sigma_m \) is an equivalent tensile flow stress representing the actual microscopic stress state in the matrix material, and \( f \) is the volume fraction of microvoids.

The microvoid volume fraction \( f \) increases during plastic deformation partly due to the growth of existing microvoids, and partly due to the nucleation of new microvoids:

\[ f = f_{\text{growth}} + f_{\text{nucleation}} \]  

The increment due to growth is given by

\[ f_{\text{growth}} = (1 - f) D_{\varepsilon} \]  

where \( D_{\varepsilon} \) is the plastic part of the macroscopic deformation rate. We employ the following equation proposed by Needleman and Rice [8]:

\[ f_{\text{nucleation}} = F_1 \frac{\sigma_m}{\sigma_0} + F_2 \frac{\sigma_{kk}}{3\sigma_0} \]
Finite element fracture analysis of WC-Co alloy

Fig. 1. Cracked WC-Co hard alloy [1].
Finite element fracture analysis of WC-Co alloy

Table I. Material constants

<table>
<thead>
<tr>
<th></th>
<th>Young's modulus E</th>
<th>Poisson's ratio v</th>
<th>Yield stress σ₀</th>
<th>Work-hardening coef. n</th>
</tr>
</thead>
<tbody>
<tr>
<td>WC-Co</td>
<td>600 GPa</td>
<td>0.22</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Co</td>
<td>200 GPa</td>
<td>0.31</td>
<td>500 MPa</td>
<td>4</td>
</tr>
</tbody>
</table>

where σ₀ is the tensile yield stress. We assume F₁ = 0.01, F₂ = 0 and the initial microvoid volume fraction, f₀ = 0.

The stress vs strain curve of matrix material is assumed to be given by

\[ \gamma = \begin{cases} \frac{\tau}{G} & \tau \leq \tau_0 \\ \frac{\tau_0}{G} \left( \frac{\tau}{\tau_0} \right)^n & \tau > \tau_0 \end{cases} \tag{6} \]

where \( \tau \) denotes the effective shear stress (\( = \sigma_\text{eq}/\sqrt{3} \)), \( \tau_0 \) the uniaxial yield strength in shear (\( = \sigma_\text{y}/\sqrt{3} \)), \( \gamma \) the total equivalent shear strain, \( n \) the strain hardening exponent, \( G = E/(1 + v) \) the shear modulus, \( E \) the Young's modulus, and \( v \) the Poisson's ratio. The material constants, \( E \), \( v \), \( \sigma_0 \) and \( n \) of Co are shown in Table 1.

The finite element mesh is shown in Fig. 4. The displacement given by the Mode-I elastic singular solution, which is characterized by the stress intensity factor \( K \), is applied to the circular boundary far from the crack tip (\( R = 12,000d \), \( d \) = depth of notch, Fig. 2). Finite element calculations based on the finite displacement theory are carried out under the assumption of the plane strain or plane stress condition.

NUMERICAL RESULTS AND DISCUSSIONS

Circumferential and hydrostatic stress

The circumferential stress \( \sigma_\theta \) on the positive X-axis is shown in Fig. 5, where \( K \approx 15 \text{ MPa}\sqrt{m} \) (= \( K_\text{c} \): fracture toughness of WC-Co). The \( \sigma_\theta \) in Co increases as the shape of Co becomes flat for the plane stress condition [Fig. 5(a)], while the \( \sigma_\theta \) in the flat Co-region decreases considerably for the plane strain condition [Fig. 5(b)].

The decrease in \( \sigma_\theta \) in the flat Co under the plane strain condition may be attributable to the surrounding elastic solid. However, the detailed description of the reason needs a careful consideration, because if we assume that a strong displacement constraint increases the hydrostatic stress \( \sigma_\text{hh} \) and hence the microvoid volume fraction \( f \) causing material softening, then it follows that the \( \sigma_\theta \) must decrease due to the softening and this is contradictory to the assumption. In fact, the \( \sigma_\text{hh} \) in the flat Co decreases under the plane strain condition, as shown in Fig. 6(b).
This problem is important for understanding the ductile fracture under a displacement constraint and, hence, will be discussed in the following two sections. Hereafter, let us concentrate on the distribution on the $X$-positive axis under the plane strain condition, and refer to it as the distribution.

**Microvoid volume fraction**

The distribution of the microvoid volume fraction $f$ is shown in Fig. 7, where $K/K_c = 1$. It is found that $f$ in the flat Co is particularly increased, and this suggests that the large value of $f$ is connected with the decrease of $\sigma_y$ and $\sigma_u$ [Figs 5(b) and 6(b)].

To examine when $f$ in the flat Co becomes high, the distributions of $f$ at various stress levels are shown in Fig. 8(a) for the flat Co and in Fig. 8(b) for the rectangular Co. It is found that $f$ in the flat Co is slightly higher than that in the rectangular Co at $K/K_c = 0.4$, while great difference appears for $K/K_c \geq 0.6$.

The rate of the microvoid volume fraction $f$ was assumed to be equal to the sum of the rates due to nucleation and growth of microvoids, as stated in eq. (3) [Section 2]. To examine whether

![Graph](image-url)
nucleation or growth is more dominant, the increments of \( f \) due to nucleation and that due to growth are shown at various stress levels in Fig. 9 and Fig. 10, respectively. It is found from these figures that the microvoid growth has a crucial effect on the increase in \( f \) in the flat Co, but the microvoid nucleation has almost no effect.

**Ductile fracture under deformation constraint**

In the last section the growth of microvoids is found to have a great effect on the ductile fracture under a strong deformation constraint. The rate of microvoid volume fraction due to growth \( \dot{f}_{\text{growth}} \) is proportional to the plastic part of macroscopic deformation rate \( D_p \) as shown by eq. (4), and the \( D_p \) is related to the hydrostatic stress \( \sigma_h \) through the constitutive equation [8].

The \( \sigma_h \) in the flat and rectangular Co at several stress levels are shown in Fig. 11(a) and Fig. 11(b), respectively. There is no such great difference in \( \sigma_h \) between the flat and rectangular
Fig. 7. Effect of Co shape on microvoid volume fraction $f$ under plane strain condition.

(a) Flat Co

(b) Rectangular Co

Fig. 8. Comparison of microvoid volume fraction $f$ in flat and rectangular Co. (a) Flat Co. (b) Rectangular Co.
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Fig. 9. Comparison of the increment of microvoid volume fraction due to nucleation $\Delta f_{\text{nucleation}}$ in flat and rectangular Co. (a) Flat Co. (b) Rectangular Co.

Co as $f$ [Fig. 8(a) and Fig. 8(b)] or in $\Delta f_{\text{growth}}$ [Fig. 10(a) and Fig. 10(b)]. Especially for $K/K_{\text{HC}} \leq 0.4$, the $\sigma_{\text{eff}}$ in the rectangular Co is slightly higher than that in the flat Co, although the difference is small. From these numerical results it is suggested that ductile fracture under strong deformation constraint proceeds in the following way.

Let us focus on damage evolution in a single Co-region. At an early stage of deformation, $\sigma_{\text{eff}}$ and hence $f$ are slightly higher under a stronger deformation constraint than under a weaker constraint, because the value of $f$ is small and the material softening is not significant (cf. Fig. 8 and Fig. 11). Figure 12(a) shows schematically this state as two points on the yield surface. It is noted that the normality rule ($(Df_{\text{nuc}}, \sigma')$) is normal to the yield surface in Fig. 11) holds because $F_2$ in eq. (2) is assumed to be zero [8]. Here $\dot{\varepsilon}_t = (2D^p_{\text{nuc}} D_{\text{eff}}^p)^{1/3}$ and $D_{\text{eff}}^p = \text{deviatoric component of the plastic part of macroscopic displacement rate}.$

Although the two points in Fig. 12(a) are close to each other, there is a small difference in direction of the normal to the yield surface between the two points. This means that $D_{\text{nuc}}^p$
and hence \( f_{\text{growth}} \) eq. (4)] in the flat Co is slightly higher than in the rectangular Co. The difference in \( f_{\text{growth}} \) is nearly equal to that in \( \tilde{f} \), because the difference of \( f_{\text{nucleation}} \) is negligible as stated above.

Since the yield surface shrinks more rapidly for greater \( \tilde{f} \), the difference in the yield surface between the flat and rectangular Co becomes larger in the next increment of load. During further loading, the state shown in Fig. 12(b) is attained, i.e. the result that even though there is almost no difference in hydrostatic stress [Fig. 11(a) and Fig. 11(b)], there exists a large difference in microvoid volume fraction \( f \) [Fig. 8(a) and Fig. 8(b)] and its increment [Fig. 10(a) and Fig. 10(b)] for \( K/K_{ic} > 0.4 \). Figure 13 is the calculated result for a point at \( X_2 = 0, X_1 = 1.0 \) at \( K/K_{ic} = 0/4 \), showing that the state in Fig. 12(b) is really attained.

As loading is further increased, the shrinkage of the yield surface of flat Co becomes prominent. It follows from this that the hydrostatic stress \( \sigma_{hk} \) begins to decrease after reaching the maximum, while the microvoid volume fraction \( f \) continues to increase. The behaviors shown in Figs 8(a) and 11(a) for \( K/K_{ic} \geq 0.6 \) and also the phenomena shown in Fig. 7

![Figure 10](image-url)
CONCLUDING REMARKS

A finite element analysis on the ductile fracture in the Co binder near the crack-tip in a WC-Co hard alloy was performed taking account of the nucleation and growth of microvoids. The ductile fracture behaviors in the Co binder or more generally under strong displacement constraint were discussed based on the numerical results. Microvoids are predicted to glow predominantly in flat Co-regions and increasing hydrostatic stresses accompany nucleation controlled state, while the subsequent growth controlled stage is related to decreasing hydrostatic stresses. In non-flat Co-regions, microvoid evolution is nucleation and growth controlled. However, it is noted that the discussion in this paper is based on $F_2 = 0$ [eq. (5)]. If $F_2$ is not equal to zero, not only does the normality rule not hold, but also the nucleation may be influenced by the displacement constraint. Further study is necessary for $F_2 \neq 0$. 
Fig. 12. States of stress in Co and change of yield surface (schematic). (a) Low load level. (b) High load level.

Fig. 13. States of stress in Co and yield surface (numerical result).

REFERENCES


(Received 3 February 1995)