Phase-stress partition and residual stress in metal matrix composites

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Abstract

Finite element (FE) modeling based on axisymmetrical cells was performed for relating the phase-stress partition and residual phase stress in metal matrix composites (MMCs) to the reinforcement volume fraction and shape, matrix hardening behavior and applied strain levels. The phase stress is defined as mean effective stresses in the constituent phases. The elastic, plastic phase-stress partition behavior during loading, and the resultant residual stress in matrix followed unloading are delineated. A set of formulas is given for predicting the value of the phase stress in each phase, and residual stress in matrix from the inclusion volume fraction and aspect ratio, as well as matrix hardening exponent and applied strain level. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

Metal matrix composites (MMCs) comprised of elastic particles or short fibers embedded in a ductile metal matrix exhibit increased stiffness, tensile strength and creep resistance relative to the matrix material. These properties derive from, and depend upon, an effective transfer of load from matrix to stiffer reinforcing second phase. It is therefore important to be able to predict the stress distribution between the two phases on loading, as well as its variation with matrix hardening behavior and inclusion geometry (volume fraction, shape and orientation). Because of its importance, an analytical formalism for load transfer through interfacial shear, known as the shear-lag theory [1], was developed in the 1950s, and has been in continuous development since then [2]. An equivalent inclusion method based on Eshelby’s theory [3,4] has been developed to predict mean phase stresses for the model system consisting of aligned ellipsoidal inclusions dispersed randomly within an isotropic matrix [5]. Using this model, the mean longitudinal and transverse internal stresses in response to applied load have been successfully described for an aligned 20% Al/SiC composites. However, it must be remembered that the Eshelby’s analysis is exact only as the volume fraction of the inclusion tends to zero. For finite inclusion it is no longer so, and the equivalent inclusion model is only an approximation, which has to be justified in light of other analytical model and numerical method. Recently, Shi et al. [6]
measured during in situ loading the lattice elastic mean phase strains in matrix and inclusion of a 15% Al/SiC composites by use of neutron diffraction. From the measured strain components longitudinal to and transverse to loading, the mean normal phase stresses were calculated from Hook’s law. The experimental results were compared with finite element (FE) analysis of a unit cell, and good agreement was found.

There is now compelling evidence that the model based on FE-analyses of a unit cell is powerful and in better agreement with experiment [7–10]. However, to date the analyses have been mainly centered on relating the flow behavior of the composites to the inclusion volume fraction [7–9], residual stress [9], reinforcement shape [7–10] and matrix hardening [7,9,10]. Relatively little attention has been paid to relate the phase-stress partition during loading to its microstructures and the properties of the component phases in MMCs.

The mean phase-stress partition during plastic deformation will result in residual phase stresses in the component phases followed unloading, which are sometime called back stress because they induce Bauschinger Effect (BE) [11–13]. It is therefore important to predict the residual phase stress from deformation levels and the microstructure of the composites for providing some in-depth insights of the BE in MMCs.

In the present study, systematic FE modeling based on axisymmetrical cells was performed for relating the phase-stress partition, as well as residual phase stress in MMCs to the reinforcement volume fraction and shape, matrix hardening behavior and applied strain levels.

2. Model formulation and calculation methods

The unit cell model shown in Fig. 1 represents the composite material idealized in terms of a uniform, periodic distribution of the particles in the matrix. Then the FE-analysis is carried out in one axisymmetric unit cell that can be regarded as an approximation to a three-dimensional (3-D) hexagonal cell. The cell is subject to an overall stress, $\sigma_c$, parallel to the cylindrical axis. Due to symmetry and periodicity, the cell must remain a right circular cylinder whose lateral surface has zero shear traction and zero normal traction and whose end surfaces have zero shear traction. The volume fraction of the inclusion, $f$, is taken as the ratio of the inclusion volume to the cell volume. The aspect ratio of the inclusion, $r$, is the same as the ratio of the height of the cell to its diameter.

The uniaxial stress–strain behavior of the matrix is characterized by

\[
\frac{\sigma}{\sigma_0} = \frac{\varepsilon}{\varepsilon_0}, \quad \varepsilon \leq \varepsilon_0
\]

\[
\sigma = \sigma_0, \quad \varepsilon > \varepsilon_0
\]

for elastic-perfectly plastic material and by

\[
\sigma = \sigma_0 \left( \frac{\varepsilon}{\varepsilon_0} \right)^n, \quad \varepsilon > \varepsilon_0
\]

for power law hardening material, $\sigma_0$ is the yield stress of the matrix, $\varepsilon_0 = \sigma_0 / E$, $E$ is the Young’s modulus of the matrix, and $n$ is the strain hardening exponent of the matrix.

The inclusion is assumed to be elastic, the Young’s modulus being 483 GPa and Poisson’s ratio $\nu = 0.165$, values chosen in investigations for SiC particles [9]. The elastic constants chosen for the matrix are those of an Al alloy, i.e., $E = 73$ GPa, Poisson’s ratio $\nu = 0.345$.

The analyses are performed by an axisymmetric unit cell using eight-noded axisymmetric FEs in a FE-program based on finite deformation theory [14]. In order to achieve high precision, the yielding surface correction [15] is used in order that the calculated stress–strain curve of the matrix exactly follows the power-law hardening defined by Eq. (2). The nonequilibrium force vectors caused
by the yielding surface correction will be considered in the next loading step by using the equilibrium correction technique [16].

As an axisymmetric cell was used in the present study, 3-D stress states of the component phases were approximated. In calculation, after each incremental loading, the stress components, $\sigma_Z$, $\sigma_r$, $\sigma_0$ and $\sigma_{ez}$ ($Z, r, \theta$ indicate axial, radial and circumferential directions of the cell, respectively), the effective stress based on Von Mises criterion at each Gaussian integral point are calculated. For such a complex stress state, it is not suitable to choose only one stress component to represent the phase stress. In the present study, the average effective stresses in the component phases are chosen as a measure of the phase stress. During tensile loading, stronger tensile stresses in inclusions will generate. The elastic recover force of the inclusion during unloading is constrained by the surrounding plastic matrix, and compressive stress is generated in the matrix. The matrix may reversely be yielding if the compressive stress is larger than the yield stress of the matrix, which will result in inelastic unloading of the composites [17]. Since the effective stress based on Von Mises criterion cannot have a negative value, we chose the stress deviator along loading axis of the cell, $S_Z = (\sigma_Z - (\sigma_Z + \sigma_r + \sigma_0)/3)$, as a criterion: a negative value of $S_Z$ indicates compression, then the effective stress is defined as negative value. This criterion, which is used to judge whether the phase stresses during unloading in tension or in compression, is suitable as compared with the experiment results [18,19]. In calculating the average values of the effective stress in two phases, the weighted averaging method was used, i.e., the stress of each Gaussian integral point of the elements are multiplied by the area occupied by this integral point, the products are summed and divided by the total area of each phase. A more detailed description of the calculation method can be found in Ref. [19].

3. Results and discussion

In order to visualize the influence of the volume fraction $f$ and aspect ratio $r$ of the reinforcement, as well as the hardening exponent $n$ of the matrix and the applied strain levels on the phase-stress partition during loading and residual stress after unloading, the following calculation patterns are carried out:

1. $n = 0, 0.1, 0.2, 0.3, 0.4, 0.5$; at fixed $f = 0.15$ and $r = 1.0$ at strain level of $\varepsilon_c/\varepsilon_0 = 6.77$.
2. $f = 0.2, n = 0.2, r = 1.0$, loading to different strain levels.
3. $r = 1.0, 2.0, 3.0, 4.0, 5.0$; at fixed $f = 0.15$ and $n = 0.2$ at strain level of $\varepsilon_c/\varepsilon_0 = 6.77$.
4. $f = 0.1, 0.2, 0.3, 0.4$; at fixed $r = 1.0$ and $n = 0.2$ at strain level of $\varepsilon_c/\varepsilon_0 = 6.77$.

3.1. Evolution of phase stress during loading and unloading

Fig. 2 shows the evolution of the phase stresses in both phases in response to applied load for the four calculation patterns described above. The following overall behaviors of the phase-stress evolution visualize from Fig. 2:

1. As expected, the stress in elastic inclusion is consistently higher due to its higher stiffness. The phase stress in each phase is linearly related to the applied load until $\varepsilon_c/\varepsilon_0 = 1$.
2. When $\sigma_{em}/\sigma_0 > 1$ (where $\sigma_{em}$ is the mean effective stress in matrix), i.e., when the matrix of the composite yields completely, the stress in elastic inclusion develops more rapidly, and is directly proportional to the applied load, while that in matrix develops with a slow rate and is nonlinearly related to the applied load.
3. In the transition range of $\sigma_c/\sigma_0 = 1 \sim \sigma_{em}/\sigma_0 = 1$, the phase stresses in both phases develop nonlinearly. The rate of the phase-stress accumulation in matrix decreases, while that in the reinforcement increases.
4. During unloading, the average effective stresses in both phases decrease, and the matrix becomes compressively loaded when the curves of the effective stress in the matrix cross the ordinate. When unloading to zero, there exist residual compressive stresses in the matrix and residual tensile stresses in the inclusion. The magnitudes of the residual stresses increase with increasing prestrain levels.
These overall phenomena of the phase-stress partition observed above agree with the FE-analysis based a unit cell composite model and diffraction measurements during loading of an Al/TiC composite to 1 pct total strain in [6], and the internal stress analysis at the mean phase-stress levels using Eshelby’s equivalent inclusion model [5].

3.2. Phase-stress partition during elastic deformation

The phase-stress partition during elastic deformation \( (\sigma_i/\sigma_0 < 1) \) is dependent upon on the elastic constant of the two phases and the inclusion aspect ratio \( r \) and volume fraction \( f \) as shown in Fig. 2. The dependencies of the elastic phase-stress partition on \( r \) and \( f \) can be drawn from Fig. 2(c) and (d) for the Al/SiC composites as shown in Fig. 3. As expected, the ratio of the mean effective stress in inclusion to that in matrix, \( \sigma_{ei}/\sigma_{em} \), increases with increasing volume fraction (Fig. 3(a)) and inclusion aspect ratio (Fig. 3(b)), which can be expressed by

\[
\sigma_{ei}/\sigma_{em} = (0.797 + 1.354f) \\
\times (1.727 + 0.881r - 0.082r^2)
\]

for the Al/SiC composites.
3.3. Yield stress of the composite

Due to stress inhomogenous in two phases, local yielding in matrix may take place even though macro applied stress \( \sigma_c \) is less than the yield stress of the matrix as shown in the FE-analyses [20,21]. The local yielding region is surrounded by elastic medium and the plastic deformation is small before macro yielding occurs. The local fluctuation in stress will not, since they average zero, give rise to a remarkable macroscopic flow [5]. Though the elastic response of the composites will slightly deviate from elastic limit due to the local yielding, macro yield strength of the composites in practice is usually defined when extensive yielding occurs. As shown in Fig. 2, when \( \sigma_{em}/\sigma_0 > 1 \), the extensive yielding occurs. Therefore, it is reasonable to define the overall yield stress of the composite, \( \sigma_{yc} \), at the point of \( \sigma_{em}/\sigma_0 = 1 \). The yield stresses of the composite varying with volume fraction \( f \) and inclusion aspect ratio \( r \) are shown in Fig. 3(a) and (b) by the solid circles. The yield stress as function of \( f \) and \( r \), which is independent of the hardening exponent \( n \) as shown in Fig. 2(a), can be expressed as

\[
\sigma_{yc}/\sigma_0 = (0.946 + 0.072r) \\
\times (1.014 + 0.390f + 4.485f^2)
\]

for the Al/SiC composites considered.

3.4. Phase-stress partition during plastic deformation

At the onset of the plastic flow in matrix, the tangent modulus of the matrix decreases, offering less resistance to deformation. The additional plastic deformation induces a plastic misfit which must be accommodated by an increased elastic strain in the reinforcement to satisfy deformation compatibility. As a result, more rapid load transfers to the stiffer reinforcement, which is proportional to the applied load when \( \sigma_{em}/\sigma_0 > 1 \) as shown in Fig. 2.

Phase-stress partition during plastic deformation in the composites is far more difficult to calculate than elastic deformation. Plastic deformation in a composite is associated with a complex slip pattern, and work hardening behavior of the matrix. It is therefore very desirable to relate the phase-stress partition during plastic deformation with the hardening behavior of the matrix, strain level of the composite, as well as the geometry of the reinforcement from the numerical results in Fig. 2.

Fig. 4 shows the variations of the normalized mean effective stresses in matrix and inclusion, \( \sigma_{em}/\sigma_0 \) and \( \sigma_{ei}/\sigma_0 \), with hardening exponent \( n \) of the matrix for the composites with 15% volume fraction inclusion \( (r = 1.0) \) at the strain level of \( \varepsilon_{c}/\varepsilon_0 = 6.77 \). At the same overall strain level, the increased hardening rate of the matrix generates higher phase stresses in both phases. However, their ratio are nearly constant, i.e., the ratio of load sharing is independent of hardening exponent of the matrix for given overall strain level of the composite.

The ratio of load sharing for given composite is dependent upon the overall strain level as shown in Fig. 5, which was drawn from Fig. 2(b). The phase stress in matrix slightly increases, while that in inclusion rises rapidly with the overall strain of the composite.

The influence of the aspect ratio \( r \) of the elastic inclusion on the phase-stress partition is shown in Fig. 6. The phase stress in the matrix approxi-
Fig. 5. Phase-stress partition and residual stress for the composites \(f = 0.2, n = 0.2, r = 1.0\) at different strain levels.

Fig. 6. Phase-stress partition and residual stress for the composites \(f = 0.15, n = 0.2\) with different inclusion aspect ratios at strain level of \(\varepsilon_c/\varepsilon = 6.77\).

Fig. 7. Phase-stress partition and residual stress for the composites \(f = 0.15, n = 0.2\) with different inclusion volume fractions at strain level of \(\varepsilon_c/\varepsilon = 6.77\).

Fig. 8. Phase-stress partition and residual stress for the composites \(f = 0.15, n = 0.2, r = 1.0\) at different strain levels.

Experimental and numerical results showed that a significant BE exists in the MMCs even without considering the intrinsic BE of the matrix material. The BE is ascribed to the residual phase stresses, which depend upon the phase-stress partition during forward loading. It is therefore significant to predict the residual phase stress resulting from plastic deformation of the composite. As shown in Fig. 5, for given composite the normalized residual phase stress in matrix, \(\sigma_{emr}/\sigma_0\), nonlinearly increases as the applied strain increases. At a given strain level, the residual stress in matrix nonlinearly increases with the hardening exponent \(n\) of the matrix (Fig. 4), and is approximately proportional to the aspect ratio \(r\) (Fig. 6) and volume fraction \(f\) (Fig. 7) of the elastic inclusion.

From the results of Figs. 4–7, the average residual effective stress in matrix, \(\sigma_{emr}\), can be predicted by

\[
\sigma_{em} = \sigma_0 (1.034 + 1.192n + 4.321n^2) \\
\times (0.926 + 0.011\tilde{\varepsilon})(1.091 - 0.604f) \\
\times (1.029 - 0.0012\bar{r}),
\]  

where \(\tilde{\varepsilon} = \varepsilon_c/\varepsilon\). As shown in Figs. 4–7, they replicate the FE-results very well.

3.5. Residual phase stress in matrix
\[
\sigma_{emr} = \sigma_0 (0.392 + 0.343n + 0.721n^2) \\
\times (1.329 + 0.567\xi - 0.014\xi^2) \\
\times (-0.044 + 3.613f)(0.241 + 0.181r)
\]

as shown by the short dot lines in Figs. 4–7.

It should be noted that the effect of the initial elastic constant of the component phases on the phase-stress partition and residual stress is negligible during plastic deformation, because the tangent modulus of the matrix during plastic flow is very small compared with that of the elastic inclusion. The results reported in Ref. [8] shows that the ratio of the Young’s modulus of the reinforcement to the matrix varies in the range of 1–10, and Poisson’s ratio of the two phases varies in the range of 0.3–3, no remarkable difference was found for the calculated plastic flow of the composites. Hence, although the expressions of Eqs. (5)–(7) are drawn from the Al/SiC system, they are applicable to most MMCs. In addition, it is also evident that although the influence rules of the parameters \(n, f,\) and \(r\) on the phase-stress partition and residual stress have been obtained by varying one and keeping the remaining parameters constant, they hold for all possible parameter combinations in the ranges considered. That is because if there exists an influence rule of a parameter in a considered model, it should be independent of a specialized case.

4. Conclusions

From the preceding analyses, the following conclusions can be drawn:

1. During elastic loading \((\sigma_c/\sigma_0 < 1)\), phase stress in each phase is linearly related to the applied load. The ratio of the loading sharing, \(\sigma_{ei}/\sigma_{em}\), is proportional to the inclusion volume fraction \(f\), and nonlinearly increases with increasing inclusion aspect ratio.

2. During full plastic loading \((\sigma_{em}/\sigma_0 > 1)\), the phase stress in elastic inclusion is directly proportional to the applied load. The load sharing ratio of the inclusion to matrix increases as the applied strain level increases. For given matrix and applied strain level, the ratio of the loading sharing, \(\sigma_{ei}/\sigma_{em}\), is proportional to volume fraction and aspect ratio of the elastic inclusion. For given inclusion morphology, the phase stress in each phase increases with hardening exponent \(n\) of the matrix. However, the ratio of load sharing is independent of \(n\).

3. In the transition range from elastic deformation to full plastic deformation of the composites, the load sharing ratio changes as the applied loading increases. The rate of the phase-stress accumulation in matrix decreases, while that in the reinforcement increases.

4. The yield stress of the composite is proportional to the inclusion aspect ratio and nonlinearly increases with increasing inclusion volume fraction.

5. For given inclusion morphology, the residual stress in matrix nonlinearly increases with hardening exponent \(n\) of the matrix, and applied strain level. For given matrix and applied strain level, the residual stress in matrix is proportional to the inclusion volume fraction and inclusion aspect ratio.

References


