Finite element modelling of Al/SiC<sub>p</sub> metal matrix composites with particles aligned in stripes—a 2D–3D comparison

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Abstract

3D Finite element calculations comparing to axisymmetric calculations have been performed to predict quantitatively the tensile behaviour of composites reinforced with ceramic particles aligned in stripes. The analyses are based on a unit cell model, which assumes the periodic arrangement of reinforcements. The results are presented in such a manner that can be directly compared for all possible aspect ratios and inclusion volume fractions. It is indicated that varying the distance between the stripes when particle volume fraction is kept constant significantly influences the overall mechanical behaviour of composites. Whereas during elastic deformation 3D and axisymmetric formulations predict quantitatively similar results, the mechanical behaviour perpendicular to the stripe direction predicted by 3D and axisymmetric models may differ depending on the inclusion volume fraction. Nevertheless an appreciable strengthening in the stripe direction independent on the model and deformation stage is predicted. © 2002 Elsevier Science Ltd. All rights reserved.

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1. Introduction

It is well known that metal matrix composites (MMCs) exhibit a significant improvement in mechanical performance over unreinforced alloys in many commercial structural applications (Christman et al., 1989; Tvergaard, 1990; Derrien et al., 1999). In recent years they also have emerged as reasonable materials for
military applications (Chin, 1999). These materials can have the highest performance in the desirable direction if continuous fibres are used. However, the utilisation of continuous reinforcements is confined by high processing costs. Composites reinforced with discontinuous fibres or particles represent a good compromise between price and performance. There exist a variety of processing techniques for the production of aluminium based composites with discontinuous ceramic reinforcements (Ibrahim et al. 1991; Harrigan, 1998). Parallel with the processing methods utilised to manufacture particulate reinforced MMCs (Lee and Sabramanian, 1992; Smagorinski et al., 1998; Tan and Zhang, 1998; Amigo et al., 2000), a number of analytical (Mori and Tanaka, 1972; Christensen and Lo, 1979; Pindera and Aboudi, 1988; Withers et al., 1989; Cristensen, 1990; Lee and Allen, 1993; Fotiu and Nemat-Nasser, 1996; Roatta et al., 1997) and numerical (Bao et al., 1991; Levy and Papazian, 1991; Llorca et al., 1991; Svobodnik et al. 1991; Zahl and McMeeking, 1991; Hom, 1992; Weissenbek and Rammerstorfer, 1993; Weissenbek et al. 1994; Zahl and Schmauder, 1994a; Zahl et al., 1994; Li et al., 1995, 1999a,b; Dong and Schmauder, 1996; Wilkinson et al., 1999; Bruzzì et al., 2001; Ji and Wany, in press) investigations of the mechanical properties of these composites have been developed. Numerical studies are usually based on the finite element technique and used for the optimisation of mechanical properties by changing of intrinsic parameters such as the inclusion volume fraction, aspect ratios and spatial arrangement of reinforcements. Typically these analyses assume a periodic distribution of reinforcements, so that a unit cell model is often employed. Though a particulate composite is generally considered as isotropic, nevertheless when applying the periodic microfield approach the simulated overall behaviour of the composite becomes anisotropic. Weissenbek et al. (1994) detailed different regular arrangements of inclusions and symmetry considerations. On the other hand, in real composites a microstructural anisotropy is frequently observed. Thus during the processing such as extrusion, ceramic particles tend to be aligned in stripes (Lee and Sabramanian, 1992; Poudens et al. 1995; Shyong and Derby, 1995; Tan and Zhang, 1998; Soppa et al., 1999) (Fig. 1). Taking into account the microstructural anisotropy, an anisotropy of mechanical properties should be expected. The purpose of the present paper is to investigate the mechanical behaviour of composites reinforced with ceramic particles aligned in stripes. We systematically study composite strengthening as a function of degree of particle alignment and inclusion volume fraction, as composite strengthening has impact for failure in quasi-brittle systems.

2. Model formulation and boundary problem

In real composites, particles have arbitrary shape and they are irregularly distributed. In our model we make an assumption that ceramic particles possess a spherical shape and they are uniformly distributed in the matrix. In that way, we use the unit cell model in the framework of the periodic microfield approach. The unit cells for three-dimensional and axisymmetric models are shown in Fig. 2a and b. A coordinate system was chosen with the origin at the particle’s centre and the z-axis coincides with the direction of particle alignment and tensile loading. The particles
Fig. 1. Microstructure of an Al/10 vol.% SiC metal matrix composite after extrusion.

Fig. 2. Schematic representation of the modelled composite: (a) cross-section along the loading axis; (b) cross-section perpendicular to the loading axis.
are represented as spheres of radius $r$, which are spaced apart by a distance $2L_Z$ in the direction of loading (Fig. 2a) and $2L_R$ (or $2L_X, L_X = L_Y$) in the transverse direction (Fig. 2b).

As proposed by Xu et al. (1999), by changing the distances $L_R$ and $L_Z$ we can change the mechanical behaviour of the composite. Therefore, the aspect ratio of the unit cell

$$ r_a = \frac{L_R}{L_Z} \quad \text{(for axisymmetric model)} \quad (1a) $$

or

$$ r_a = \frac{L_X}{L_Z} = \frac{L_Y}{L_Z} \quad \text{(for 3D model)} \quad (1b) $$

becomes the important parameter which defines the degree of particle alignment in such an idealised composite. If $L_R(L_X)$ is greater than $(r_a > 1)$ we can model the loading\(^1\) along the stripe direction (longitudinal loading) and when $L_R(L_X)$ is less than $L_Z$ $(r_a)$ we suppose that the loading is perpendicular to the stripe direction (transverse loading). It should be noted here, that the sizes of axisymmetric and 3D unit cells do not coincide. We keep constant the particle volume, the volume of unit cell and the cell aspect ratio, but the sizes of the cells for 2D and 3D calculations differ from each other.

The volume fraction of the particles is related to the volume of the unit cell through particle volume fraction by

$$ \frac{4}{3} \pi r^3 = \pi L_R^2 (2L_Z) f_v \quad (2a) $$

for the axisymmetric model, and by

$$ \frac{4}{3} \pi r^3 = 4L_X^2 (2L_Z) f_v \quad (2b) $$

for the 3D model, where $r$ is the radius of the particles and $f_v$ is the particle volume fraction. Using the definition of cell aspect ratio (1), one can evaluate the minimal and the maximal values of cell aspect ratio for a given particle volume fraction. They are given as

$$ r_a^{\text{max}} = \lim_{r \to L_Z} \frac{2r^3}{3L_Z^2 f_v} = \sqrt[3]{\frac{2}{3f_v}} \quad (3a) $$

$$ r_a^{\text{min}} = \lim_{r \to L_R} \frac{3L_R^3 f_v}{2r^3} = \frac{3f_v}{2} \quad (3b) $$

for axisymmetric and

\(^1\) Here and further in the next we suppose that loading is specified as a tensile loading in the Z direction.
for 3D models. Thus, when investigating the influence of cell aspect ratio on the overall composite behaviour for a given particle volume fraction it should be kept in mind that we are restricted by minimal and maximal values of the cell aspect ratio (see Fig. 3\(^2\)) defining the degree of particle alignment.

For the analyses the finite element code ABAQUS (Anon., 1998) is employed using six-noded triangular axisymmetric elements and ten-noded tetrahedral elements, respectively.

\[^{2}\text{In this figure and in Figs. 5–8 the x-axis left to 1 has inverse proportionality. Therefore instead of linear dependency, given by (3b) and (4b) we have reverse dependency for cell aspect ratios less than 1. This has been made in order to show the loading along and perpendicular to the stripe direction at the same scale in Figs. 5–8.}\]
Uniaxial tensile loading is incrementally applied by displacements, so the boundary conditions are:

\[
U_Z(x, y, L_Z) = U; \quad U_X(0, y, z) = 0; \quad U_Y(x, 0, z) = 0; \quad U_Z(x, y, 0) = 0 \tag{5a}
\]

for the 3D model and

\[
U_Z(R, L_Z) = U; \quad U_R(0, z) = 0; \quad U_Z(x, 0) = 0 \tag{5b}
\]

for the axisymmetric analysis. Since the shape of the unit cell must remain a right (straight-sided) parallelepiped in 3D calculations or a right circular cylinder in the axisymmetric model for all deformation states, the following additional constraints are essential:

\[
U_X(L_X, y, z) = U_X(L_X, 0, 0);
U_Y(x, L_Y, z) = U_Y(0, L_Y, 0);
U_X(L_X, L_Y, 0) = U_Y(L_X, L_Y, 0) \tag{6a}
\]

Fig. 4. Typical finite element mesh used for axisymmetric analysis (inclusion volume fraction is 10%).
for 3D and
\[ U_X(L_X, z) = U_X(L_X, 0) \]  \hspace{1cm} (6b)

for axisymmetric models.

During the analysis true macroscopic stresses and strains are calculated. The true macroscopic axial stress is evaluated as an average of local stresses over the matrix and the inclusion. The true macroscopic strain is defined as:
\[ \varepsilon_{zz} = \ln(1 + U_Z/L_Z) \]  \hspace{1cm} (7)

In this study we assume that the composite material is initially stress-free, so residual stresses due to processing are not taken into account. In order to avoid discrepancies caused by different finite element meshes on the inclusion-matrix interface, in our calculations we always use the same inclusion with invariable mesh. The mesh

<table>
<thead>
<tr>
<th>Material</th>
<th>( E ) (GPa)</th>
<th>( \nu ) (–)</th>
<th>( \sigma_0 ) (MPa)</th>
<th>( \sigma_m ) (MPa)</th>
<th>( \varepsilon_c ) (–)</th>
<th>( m ) (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al</td>
<td>73.0</td>
<td>0.345</td>
<td>383.0</td>
<td>454.0</td>
<td>0.048</td>
<td>47.0</td>
</tr>
<tr>
<td>SiC</td>
<td>483.0</td>
<td>0.165</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
size is chosen in such a way that subsequent refinement does not affect much the stress-strain behaviour. A typical finite element mesh is shown in Fig. 4.

3. Description of inclusion and matrix materials

In our model we consider an elastic silicon carbide (SiC) inclusion and elastic-plastic aluminium (Al) matrix. The bonding between the inclusions and the matrix is assumed to be perfect, debonding on the inclusion-matrix interface is not studied. Conventional $J_2$-flow theory is employed and matrix yielding obeys the von Mises yield criterion. The plastic constitutive relations are taken in the Voce-type form with parameters fitted to the experimental data of Al 6061 alloy (Wulf et al., 1994; Wulf, 1995):

$$\sigma = \sigma_0 + (\sigma_m - \sigma_0)(1 - e^{-\frac{\varepsilon}{\bar{\varepsilon}}} + m\varepsilon)$$

(8)

Fig. 6. Composite strengthening with respect to matrix material at overall strain 0.001 given by axisymmetric model.
where \( \sigma_0 \) and \( \sigma_m \) are the flow- and the saturation stress respectively, \( \varepsilon_c \) is a normalising characteristic plastic strain, \( m \) is a parameter responsible for additional linear hardening. Young’s modulus \( E \), Poisson’s ratio \( \nu \) and parameters from Eq. (8) are adduced in Table 1.

4. Results and discussion

The macroscopic response to uniaxial tension for the particle-reinforced composites with 3, 5, 10, 15 and 20% inclusion volume fraction has been modelled. The true stress-strain curves obtained by the axisymmetric model for different cell aspect ratios are given in Fig. 5 for the particle volume fraction 3%. The dashed curves correspond to the loading along the stripe direction and the dotted curves correspond to the loading perpendicular to the stripe direction. It is clearly seen that the composite strengthens with increasing cell aspect ratio and softens with decreasing cell aspect ratio. In other words, the larger the distance between the stripes (or the more the

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**Fig. 7.** Composite strengthening with respect to matrix material at overall strain 0.001 given by three dimensional model.
degree of particle alignment\(^3\) the more the composite strengthens in the stripe direction and the more it softens in the direction transverse to the stripe.

Bao et al. (1991) proposed to describe the composite strengthening in terms of the asymptotic reference stress\(^4\) because for the case of elastic inclusions perfectly bonded to the elastic–plastic matrix the composite will necessarily harden at the same rate as the matrix at sufficiently large strains. As pointed out by Bao et al. (1991), Zahl et al. (1994) and Dong and Schmauder (1996), the calculation of the asymptotic reference stress requires strains that are as much as 10 times \(\varepsilon_0\) (yield strain). However, at these strains damage and failure mechanisms could already play an essential role. Therefore, we suggest to study the strengthening effect simply in terms

\(^3\) By “degree of particle alignment” we imply the degree of microstructural anisotropy, which is indicated by cell aspect ratio \(r_a\). The lowest degree of particle alignment corresponds to \(r_a = 1\), the remoter from 1, the higher the degree of particle alignment. Thus, for instance, composites with \(r_a = 2\) and \(r_a = 1/2\) are assumed to possess the same degree of particle alignment.

\(^4\) The composite reference stress is calculated by normalising the composite stress by the matrix stress at the same strain value. At sufficiently large strain, the value of composite reference stress will converge to a constant, named asymptotic reference stress.
of the macroscopic axial composite stress normalised by the average matrix stress at the same strain value. We have chosen two representative strain values: \( \varepsilon = 0.001 \) which represents purely elastic behaviour (Figs. 6 and 7) and \( \varepsilon = 0.02 \) which represents fully plastic behaviour (Figs. 8 and 9). As has been mentioned above, the graphs in Fig. 6–9 are not usual, because the X-axis is not linear. It has inverse proportionality in region I (left to 1) which corresponds to the case of loading perpendicular to the stripe direction. The region II is the case of loading along the stripe direction. From Figs. 6 and 7 one can see that while the composite deforms elastically, an increase of cell aspect ratio results in a composite strengthening. Besides, the strengthening along the stripe direction is more pronounced than the softening perpendicular to the stripe direction. Comparing results from axisymmetric unit cell model (Fig. 6) with those from 3D calculations (Fig. 7), it can be concluded that axisymmetric models predict higher composite strengthening than full 3D analyses.

The strengthening curves obtained by the axisymmetric analysis at the plastic regime of deformation (Fig. 8) show a similar behaviour as those obtained at the elastic stage of deformation for small inclusion volume fractions (3–10%). Slight softening with decreasing cell aspect ratio in region I and a significant strengthening

![Graph showing composite strengthening with respect to matrix material at overall strain 0.02 given by three dimensional model.](image)

**Fig. 9.** Composite strengthening with respect to matrix material at overall strain 0.02 given by three dimensional model.
with increasing cell aspect ratio in region II are observed. However, in region I the behaviour changes: the composite commences to strengthen with decreasing cell aspect ratio when the particle volume fraction is 15% and higher. Such behaviour could develop due to the invariance of volume of the matrix beyond the yield point. The matrix volume is kept constant and as a consequence the Poisson ratio changes from 0.345 to 0.5. Thus, the influence of neighbouring particles on each other is significantly increased. But on the other hand, when the full 3D analysis is applied the composite does not strengthen with decreasing cell aspect ratio even for volume fraction 20% (Fig. 9) and moreover it softens as at the elastic stage. This could be explained in such a way, that the particle in a cylindrical cell is influenced by an infinity of transverse neighbouring particles because it has a neighbouring particle in any transverse direction around its circumference, while the particle in a 3D simple cubic unit cell has only 4 nearest transverse neighbours. In that way, when the particle volume fraction is large enough and the cell aspect ratio is such that neighbouring particles are close to each other, the results of finite element calculations with axisymmetric models may show a quite different behaviour compared to results obtained by a full 3D analysis. Thus, we conclude that an axisymmetric model overestimates the coupling between neighbouring particles.

Fig. 10 shows the comparison of our numerical results with experimental data for the Al(6061) alloy and Al(6061)/SiC(20%) obtained by Wulf (1995). Axisymmetric and 3D analyses give the same stress–strain curves for the pure matrix and agree well with the experimental curve. For composites with 20% volume fraction of silicon carbide, the axisymmetric analysis predicts a higher stress-strain curve than full 3D analysis. The results of 3D simulation agree much better with the experiment and is, therefore, more appropriate for modelling of real materials.

Fig. 10. Experimental stress–strain curves (by Wulf, 1995) for Al(6061) alloy and Al(6061)/SiC(20%) metal matrix composite in comparison with our numerical stress–strain curves.
In Figs. 6–9 a curve with higher particle volume fraction is always above a curve with lower particle volume fraction, so the composite strengthens with increasing inclusion volume fraction. Comparing Figs. 6 and 8 on the one hand and Figs. 7 and 9 on the other, it is seen that when the composite deforms elastically, the values of normalised stress are higher than those calculated at the plastic stage. However, it should be kept in mind, that at the same time elastic stress values are much less than stress values calculated during plastic deformation.

5. Conclusion

The overall mechanical behaviour of composites with different degree of particle alignment has been investigated using three dimensional and axisymmetric unit cell models. The results show that varying the distance between the stripes when the particle volume fraction is kept constant can considerably change the overall mechanical behaviour of composites. Namely:

- an obvious improvement of mechanical properties in the stripe direction independent of deformation stage has been found;
- for the case of elastic deformation the raise of degree of particle alignment results in a strengthening behaviour (elevation of stiffness) along the stripe direction and results in a softer behaviour (reduction of stiffness) when loading perpendicular to the stripe direction;
- on the stage of plastic deformation the composite behaviour in axisymmetric and 3D formulations becomes different. On the one hand, when loading in the stripe direction the higher the degree of particle alignment the more the composite becomes strengthened. But on the other hand, when loading perpendicular to the stripe direction the 3D analysis predicts composite softening with increasing the degree of particle alignment for all inclusion volume fractions, but in contrast the axisymmetric analysis may predict strengthening by increasing the degree of particle alignment for inclusion volume fractions higher than 15%;
- nevertheless, for all cases the composite necessarily strengthens with increasing the inclusion volume fraction.

It should be noted here that our model assumes a periodic arrangement of inclusions and due to the absence of failure the numerical results tend to overestimate the stress-strain response of real composites. The presented results allow us to assume that the axisymmetric unit cell overestimates the coupling between neighbouring reinforcements and tend to overconstrain plastic flow in the matrix. The predictions made with three-dimensional models are seemed to be more realistic.

5 Figs. 5 and 6 represent elastic behaviour of composites (overall strain is 0.001), Figs. 7 and 8 represent plastic behaviour of composites (overall strain is 0.02). This can be seen, for instance, from Fig. 4.
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References