Numerical simulation of intermittent yielding at the macro and mesolevels

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Abstract

Presented in this paper is the simulation of intermittent yielding phenomena experimentally observed in aluminum and copper alloys. An earlier developed macro–meso yield criterion is modified to take into consideration multiple shear band propagation. Calculations of tension of an Al6061 alloy test piece as an example were performed to investigate the serrated character of the stress–strain curve. The dynamics of stress and strain patterns on the macro and mesoscale levels is studied in details.

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1. Introduction

The phenomenon of intermittent yielding in aluminum and copper alloys has been well studied experimentally, for instance in [1–5], and attributed to the consecutive periodical formation of shear bands at the mesoscale. As a special case of such an anomalous behavior, Lüders band propagation is characterized by single displacement of macroscopic localization zone along the test piece. There are some physically based attempts to simulate PLC band propagation and intermittent yielding [6–10].

Earlier Lüders band origination and propagation was numerically investigated using 20MnMoNi55 steel as an example [11]. In order to describe localized plastic yielding, we formulated a specific yield criterion, assuming for any local region to be plastically deformed, provided the equivalent plastic strain reaches its critical value in one of the nearest local regions. Such a phenomenological approach combines numerical techniques of continuum and discrete mechanics, particularly...
the finite-difference approximation [12] and the method of cellular automata [13].

In this paper this approach is extended to the case of multiple shear band generation and propagation. For so doing, we modify the yield criterion developed earlier in [11], with taking into account periodical origination of shear bands in the vicinity of a clamp and their step-by-step propagation throughout the specimen as loading continues. The mathematical description of the 2D-problem in its dynamic formulation and the yield criterion are given in Section 2.

Using the model developed, two-dimensional calculations were carried out for the aluminum alloy Al6061 as an example. Model parameters for the alloy were derived from the experiment on tensile loading [1].

Computational results and their analysis based on a mesomechanical point of view are presented in Section 3. Physical mesomechanics [13,14] considers a solid under loading as a multilevel self-organized system of different scale levels. Within this concept, plastic deformation is treated as shear stability loss that occurs in a self-consistent way at the micro-, meso-, and macroscale levels. Therefore, analyzing the computational results, we have paid special attention to the examination of the interconnection between elasto-plastic patterns on the meso- and macroscale levels. The computational results show good agreement with those observed experimentally in [1].

2. Mathematical formulation

The physical problem of periodical shear band propagation along the test piece can be reduced to a two-dimensional statement. In this paper the phenomenon of intermittent yielding in aluminum alloys was simulated within the plane strain formulation. Numerical solutions were performed in terms of Lagrangian variables using the finite-difference method [12].

The total system of equations includes: the equations of motion and continuity, the expressions for components of strain rate tensor and the constitutive equations. Special attention is given to the plasticity criterion definition.

Let us write the equations of motion as follows:

\[ \frac{\partial \sigma_{11}}{\partial x} + \frac{\partial \sigma_{21}}{\partial y} = \rho \frac{\partial v_1}{\partial t}, \quad \frac{\partial \sigma_{12}}{\partial x} + \frac{\partial \sigma_{22}}{\partial y} = \rho \frac{\partial v_2}{\partial t}, \]  

where \( \rho \) is the mass density, and \( v_1 \) and \( v_2 \) are the components of particle velocity vector.

The mass conservation law takes the form

\[ \frac{\partial \rho}{\partial t} + \rho \left( \frac{\partial v_1}{\partial x} + \frac{\partial v_2}{\partial y} \right) = 0. \]

In the case of plane strain there exist three independent non-zero components of strain rate tensor

\[ \begin{align*}
\frac{\partial e_{11}}{\partial t} &= \frac{\partial v_1}{\partial x} + \frac{\partial v_2}{\partial y}, \\
\frac{\partial e_{22}}{\partial t} &= \frac{\partial v_2}{\partial x} - \frac{\partial v_1}{\partial y}, \\
\frac{\partial e_{12}}{\partial t} &= \frac{1}{2} \left( \frac{\partial v_1}{\partial y} + \frac{\partial v_2}{\partial x} \right). 
\end{align*} \]

Resolving the stress tensor into spherical and deviatoric parts \( \sigma_{ij} = -P \delta_{ij} + S_{ij} \), the following constitutive equations for the stress deviators and pressure are obtained

\[ \begin{align*}
\frac{\partial S_{11}}{\partial t} &= 2\mu \left( \frac{\partial e_{11}}{\partial t} - \frac{1}{3} \frac{\partial e_{kk}}{\partial t} \right) - \frac{\partial}{\partial t} \lambda S_{11}, \\
\frac{\partial S_{22}}{\partial t} &= 2\mu \left( \frac{\partial e_{22}}{\partial t} - \frac{1}{3} \frac{\partial e_{kk}}{\partial t} \right) - \frac{\partial}{\partial t} \lambda S_{22}, \\
\frac{\partial S_{33}}{\partial t} &= 2\mu \left( -\frac{1}{3} \frac{\partial e_{kk}}{\partial t} - \frac{\partial}{\partial t} \lambda S_{33} \right) = -\left( \frac{\partial S_{11}}{\partial t} + \frac{\partial S_{22}}{\partial t} \right), \\
\frac{\partial S_{12}}{\partial t} &= 2\mu \left( \frac{\partial e_{12}}{\partial t} - \frac{\partial}{\partial t} \lambda S_{12} \right), \\
\frac{\partial P}{\partial t} &= -K \frac{\partial e_{kk}}{\partial t},
\end{align*} \]

where \( K \) and \( \mu \) are the bulk and the shear moduli, respectively, and \( \delta_{ij} \) is the Kronecker delta. \( \lambda \) is a scalar parameter, following the plasticity theory law \( \frac{\partial e^p}{\partial t} = \frac{\partial}{\partial t} S_{ij} \) associated with the yield criterion \( \sigma_{eq} = f(e_{eq}, \sigma_0) \).

Here the function \( f(e_{eq}) \) prescribes strain hardening, \( \sigma_0 \) is the yield point, \( \frac{\partial e^p}{\partial t} \) is the plastic strain rate tensor, and \( \sigma_{eq} \) and \( e_{eq} \) are the equivalent stress and plastic strain, respectively:

\[ \sigma_{eq} = \frac{1}{\sqrt{2}} \left\{ (S_{11} - S_{22})^2 + (S_{22} - S_{33})^2 + (S_{33} - S_{11})^2 + 6(S_{12}^2 + S_{23}^2 + S_{31}^2) \right\}^{1/2} \]

(6)
and

\[
\varepsilon_{eq} = \frac{\sqrt{2}}{3} \left\{ (\varepsilon_{11}^p - \varepsilon_{22}^p)^2 + (\varepsilon_{22}^p - \varepsilon_{33}^p)^2 + (\varepsilon_{33}^p - \varepsilon_{11}^p)^2 \right\} + 6 \left( \varepsilon_{12}^p + \varepsilon_{23}^p + \varepsilon_{31}^p \right) \frac{1}{2},
\]

(7)

The conventional continuum mechanics formulation described above does not allow to model shear band propagation at the mesolevel. Earlier we investigated numerically Lüders band origination and propagation, using 20MnMoNi55 steel as an example. The classical force yield criterion (5) was modified, assuming any local region \( D \) to be plastically deformed, provided the equivalent plastic strain reaches its critical value in one of the nearest local regions \( D^* \):

\[
\varepsilon_{eq}^* = \varepsilon_0,
\]

(8)

where \( \varepsilon_0 \) is the threshold value of plastic strain. Thus, the newly developed criterion covers both (5) and (8) rules. This combined formulation presumes that the onset of plastic flow in a local region depends not only on its stress–strain state parameters but also on those in the adjacent material. This statement of the problem offers a means to describe propagation of localized shear bands, this being its advantage over the classical approach.

In this paper the approach is developed as applied for modeling periodical shear band generation and propagation. The experimental data on Portevin–Le Chatelier effect in aluminum and copper alloys testify that the multiple shear band propagation results in a serrated character of the stress–strain curve. Every serrated jump corresponds to the formation of a certain shear band. As a rule, both the amplitude of serrated jumps and the value of the strain between two jumps following each other increase as the strain hardening develops. Taking this fact into account, we modify Eq. (8) to

\[
\varepsilon_{eq}^* = \varepsilon^*(\zeta, \varepsilon_0), \quad \text{where} \quad \zeta = f(\varepsilon_{eq}, \sigma_0)/\sigma_0.
\]

(9)

Shear bands generate periodically near the clamps, provided \( \Delta \varepsilon_{eq}^{\text{min}} = \varepsilon^*(\zeta, \varepsilon_0) \). Here \( \Delta \varepsilon_{eq}^{\text{min}} \) is the minimal increment of equivalent plastic strain resulting from the previous shear band propagation through the region under study. By analogy with \( f(\varepsilon_{eq}, \sigma_0) \), the functions \( \varepsilon^*(\zeta, \varepsilon_0) \) and \( e(\zeta, \varepsilon_0) \) are of a kind of limiting surfaces in space of strains. The simple ratios \( \varepsilon^*(\zeta, \varepsilon_0) = \varepsilon_0 \exp \left( \frac{\zeta}{\varepsilon_0} \right) \) and \( e(\zeta, \varepsilon_0) = \varepsilon_0(\zeta - 1) \) were derived during computational tension tests for the aluminum alloy Al6061.

3. Computational results: Analysis and discussion

Calculations were performed for the aluminum alloy Al6061 which demonstrates intermittent yield behavior [1]. The test-piece under calculation is presented in Fig. 1. In this work we investigate the deformation of a rectangular region. According to the plane strain formulation the region corresponds to the lateral face of a flat test piece experimentally investigated under tension. Considering a domain \( D(x, t) \) with the boundary \( B(x, t) \), where \( x \) is the radius vector of the continuum point and \( t \) is the time, the initial conditions at \( t = 0 \) for every \( x \in D(x, 0) \) take the form:

\[
v_i(x) = 0, \quad \sigma_{ij}(x) = 0, \quad \rho(x) = \rho^0(x), \quad (i, j = 1, 2).
\]

(10)

Boundary conditions on the right and left surfaces of the region under calculation simulate the clamp displacement with a constant speed, while on the top and bottom surfaces they correspond to the free surface and symmetry conditions, respectively:

\[
\begin{align*}
&v_1(x, t) = \text{const.} = -V & \text{for} & t \geq 0, & x \in B_1, \\
&\sigma_{ij}(x, t) \cdot n_j = 0 & \text{for} & t \geq 0, & x \in B_2, \\
&v_1(x, t) = \text{const.} = V & \text{for} & t \geq 0, & x \in B_3, \\
&v_2(x, t) = 0 & \text{for} & t \geq 0, & x \in B_4,
\end{align*}
\]

(11)

where \( B = B_1 \cup B_2 \cup B_3 \cup B_4 \) and \( n_j \) denotes vector components normal to the surface.

![Fig. 1. Schematic of the region under study. Initial and boundary conditions.](image-url)
To describe strain hardening use was made of the exponential function:

\[ f(\varepsilon_{eq}) = A - B \exp\left(-\frac{\varepsilon_{eq}}{C}\right) \text{ MPa}, \]

where

\[ \sigma_0 = f(0) = A - B. \]

The values of the parameters \( A, B \) and \( C \) chosen from the experimental data [1], elastic moduli and the density given in [15] as well as the critical value of plastic strain derived from the computations for the aluminum alloy are presented in Table 1.

The calculation results presented in Figs. 2–4 demonstrate essentially irregular stress and strain behavior. Fig. 2 shows the integral stress–strain diagram (a) and its parts plotted in details (b)–(d). The stress was calculated as an average value of equivalent stress over the mesovolume:

\[ \langle \sigma_{eq} \rangle = \frac{\sum_{i=1}^{n} \sigma_{eq}^{i} s^{i}}{\sum_{i=1}^{n} s^{i}}, \]

where \( n \) is the number of computational mesh points, \( s^{i} \) is the local volume, the value of which was taken to be \( 10^{-4}\text{cm}^{2} \) in the calculations. The strain represents relative elongation of the region along the \( X_1 \) direction.

First, the specimen undergoes uniform elastic deformation, that corresponds to the linear monotonous part of the stress–strain curve (see Fig. 2(a)). As tension continues, the shape, amplitude and period of the serrated jumps change due to the development of strain hardening. Presented in Fig. 2(b)–(d) are the parts of the stress–strain curve, corresponding to different values of elongation with a higher resolution in time. At the very beginning of plastic yielding (Fig. 2(c) and (d)), a series of isolated pulses are observed on the curve. Each pulse corresponds to the band formation and propagation from one end of the specimen to another. A steady part between each two jumps corresponds to quasi-uniform deformation throughout the specimen and indicates the period between the origination of the bands. The plastic fronts propagate from one grip to the other at an approximately steady speed. A comparison of curves in Fig. 2(b) and (c) shows that the more the specimen is elongated, the lower the velocity of band propagation.

At the later stage of elongation (Fig. 3) due to both the strain hardening and non-homogeneous stress–strain distribution the movement of shear bands becomes irregular, i.e. the band velocity increases and decreases (up to full stop in Fig.

<table>
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<th>Table 1</th>
<th>Parameters of the model for the Al6061</th>
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<td>( K ) [GPa]</td>
<td>( \mu ) [GPa]</td>
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<td>200</td>
<td>26</td>
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4(l)–(n)) alternately. Such a non-uniform motion of the plastic fronts results in oscillations in the stress–strain curve (Fig. 2(b)). The oscillations with higher amplitude of the envelope correspond to the case when only one band forms near one of the grip and then propagates towards the opposite end, while the lower amplitude of the envelope indicates origination of the bands near both grips and their alternative or simultaneous motion towards each other. This is because two or more bands together may produce even more strain resulting in deeper stress drop than only one band. The same conclusion in respect to the amplitude value is true for the case of the isolated pulses (Fig. 2(d)).

Let us consider a segment of deformation on the mesoscale level in more detail (Fig. 4). Initially plastic deformation originates in the vicinity of the right grip. At the very beginning, the plastic front propagates perpendicular to the direction of tension (Fig. 4(h) and (i)). Moving away from the grip, the front exhibits a tendency to deviate from its initial orientation, inclining towards the axis of tension. From the mathematical point of view, this phenomenon is clearly explained by a difference in stress–strain conditions near the free surface and inside the specimen. Indeed, under the plane strain state the component of stress tensor normal to the
free surface is equal to zero on the surface and non-zero in the bulk of the material that, according to Eq. (6), leads to a higher level of equivalent stresses near the surface than that in the volume. Thus, the yield criterion in the surface areas is fulfilled at lower external forces than that in corresponding volume points.

4. Summary

In this paper we presented a mesomechanical model which describes the phenomenon of intermittent yielding in the aluminum alloy Al6061. The yield criterion developed in our earlier work has been modified and applied in the simulation of multiple generations of plastic flows in the vicinity of stress concentrators and their step-by-step propagation throughout the specimen. The dynamics of propagation of the plastic fronts on the mesoscale level and its influence on the macroscopic behavior is investigated. The interrelation between material elasto-plastic responses on the meso- and macroscale levels is studied in details.

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References


