Modelling of Mesomechanics of Portevin-Le Chatelier Effect by Relaxation Element Method
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**Summary**

The results of 2D simulation of strain localizations on the meso-level in the surface layer of a polycrystal with a highly pronounced Portevin Le Chatelier (PLC) effect are presented. For such a purpose the Relaxation Element Method (REM) has been used. A fundamental property of solid “plastic deformation is accompanied by stress relaxation in local volumes of a loaded solid” lies on the basis of this method. The elaborated REM model operates on the principle of cellular automata. The results are in good qualitative agreement with known experimental data.

**Introduction**

The description and modelling of the localization of plastic deformation requires accounting for multilevel character of the processes development in the deformed system, where the near-surface layer is a sovereign subsystem. The evolution of inhomogeneous distribution of plastic deformation is realized under the action of various stress concentrators and first of all under the operation of the stress concentrators at the free surface. The pattern of macrolocalization of plastic deformation inherits the character of its distribution in the thin near-surface layer.

The low shear stability of the near-surface layer causes the development of shears in it along the conjugate direction of maximum tangential stresses. The coupling of near-surface layer with substrate causes periodical arising of transverse bending in it and stress connected with them. The later relax with generation of the bands which cause intermittent flow of deformed specimen.

Such a scheme of a deformation of solid as a multilevel system allows to use for the simulation of intermittent flow relaxation element method REM [3]. The fundamental property of solid: “plastic deformation is accompanied by stress relaxation in the local volumes of loaded solid” is laid on the basis of the method. The details of the surface shears along the direction of maximum shear stresses can be described within the framework of plane-stress model, and the influence of the coupling between the surface layer with substrate defines the conditions for the evolution of surface shears.

Simulation of the intermittent flow in such a statement of problem is performed in the present paper.

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The results of two-dimensional simulation of strain localization in poly-crystal with highly pronounced Portevin Le Chatelier effect (PLC) on mesoscale level is presented.

The plane-stress case, corresponding to the stress-strain state of a plane poly-crystal under tensile loading is considered.

**Elementary site of plastic deformation of pure shear**

On a mesoscopic level a separate grain of polycrystal, undergone plastic deformation to some extend can be simulated in the form of the site of plastic deformation of the round shape. Plastic deformation of a local deformed matrix due to the relaxation nature of plastic deformation is accompanied y decreasing in elastic energy in the given region. The stress-strain state of the system for the case when the stress drop in a value \( \Delta \sigma \) takes place in the round region under the external tensile stress is considered [1]. Stress relaxation in the value \( \Delta \sigma \) takes place as a result of plastic deformation on according to the scheme of pure shear in the conjugate directions at an angle of 45° to tensile axis. By relaxation element method for the given case, the analytical expressions for the whole components of the tensor of the field of internal stresses in the plane with the prescribed site of plastic deformation under uniaxial tensile loading along \( Oy \)-axis have been derived. The distribution of the shear stress in the conjugate directions at an angle of 45° to tensile axis is described by the formulae

\[
\tau(x,y) = \Delta \sigma \left\{ \begin{array}{l}
\frac{(\beta+1)^2}{2(\beta+3)} \left[ \frac{3(\beta+3)^2}{(\beta+3)^2} - 2 \right] \left[ 1 - 8\left(1 - y^2/r_2^2\right)y^2/r_2 \right], \quad if \quad r^2 \geq a^2 \\
-0.5 + \left(\frac{\beta}{2(\beta+3)(\beta+3)}\right)^{\beta+1} \left[ \frac{\beta^2-1}{2(\beta+3)^2} \left[ 1 - 8\left(1 - y^2/r_2^2\right)y^2/r_2 \right] + 0.5 \right], \quad if \quad r^2 \leq a^2 
\end{array} \right.
\]

(1)

where \( a \) - is the radius of the site of crystallite, \( r \) - is the distance from the center of crystallite to the point with the coordinate \( \beta \) is the parameter which defines the value of the gradient of plastic deformation near the boundary of the site. The field of plastic deformation is characterized by the tensor with the components.

\[
\varepsilon_y(x,y) = 2\frac{\Delta \sigma}{E} \left[ 1 - \left( \frac{r}{a} \right)^{\beta+1} \right], \quad \varepsilon_x(x,y) = -\varepsilon_y(x,y), \quad \varepsilon_{xy}(x,y) = 0,
\]

(2)

The spatial distribution of the given stress is represented in Fig. 1a for the value \( \beta =6 \). Contour plot of the shear stress is depicted in Fig. 1b. It is seen that under the condition of plastic deformation of pure shear, the inhomogeneous stress field arises with elevated stresses along the directions of 45° with respect to the tensile axis. That means that first of all those crystallites will be involved into plastic deformation, which are located along the pointed direction. The represented example of the site of plastic deformation have been used as the basic relaxation element,
defining the sequence of the separate crystallite involvement into plastic deformation.

Figure 1: Field of stresses (a) and isoline (b) of pure shear in conjugate directions at an angle of 45° to tensile axis for the site of plastic deformation.

**Modelling of interrupted flow**

The proposed model operates on the principle of cellular automata. The calculational field is represented in the form of the matrix 10x50 points—the centers of crystallites. Onset of plastic deformation was initiated by the relaxation element, represented in Fig. 1 and equations (1) and (2), which was placed at the edge of the calculated field. This corresponds to the plastic deformation (2) of a separate grain, exposed to the edge of polycrystal. Given grain causes inhomogeneous stress field (1) in the volume of polycrystal. The minimum external stress was calculated at which in the center of any crystallite the shear stress attained the critical value \( \tau = 50 \) MPa according to Tresca criterion. In the corresponding point, a new relaxation element of considered type was placed, simulated plastic deformation (2) of a new crystallite, which changed the stress state in the volume of polycrystal. Further, the procedure of definition of the coordinate of the point, where, fist of all, a critical value \( \tau_{cr} \) is achieved and putting in it new relaxation element were repeated many times.

Thus, the algorithm of the model follows the following procedure, which is described in details in [2]. For each crystallite the value of external applied stress, under which the critical shear stress is achieved, is calculated. At \( n^{th} \) step in arbitrary crystallite with the \((x, y)\) coordinate the value \( \tau_{cr} \) is achieved at the external stress

\[
\sigma(x, y) = \frac{2(\tau_{cr} - \sum_{i=1}^{n-1} \Delta \tau(x_i, y_i))}{1 + 2\Delta \tau(x_n, y_n)/\Delta \sigma}.
\]

Here, the sum defines the contribution from the previous relaxation elements to the shear stress. The minimum magnitude of this function \( \sigma(x, y)_{min} \) corresponds to the minimum external stress at which there is a possibility of involvement of a
single crystallite into plastic deformation. The coordinates of the point in which \( \sigma(x,y)=\sigma(x,y)_{\min} \) and a new relaxation element was placed there. Thus, given crystallite received the discrete portion of plastic deformation (5) and corresponding field of internal stresses (4). As a relaxation element, crystallite influences the field of stresses of a whole volume of solid. Interaction of the fields of internal stresses from the crystallites, undergone plastic deformation together with external applied stress results in the formation of the structures of localized plastic deformation.

The equation (1) is valid for the infinite plane. At that time at the lateral edges of calculational field, simulating the geometry of the specimen, the normal and tangential stresses are present. These stresses should be absent according to the boundary conditions of loading. Removing of these stresses have been performed by boundary element method [3]. By doing that, the corresponding changing of internal stress field of the simulated polycrystal was taken into account.

The assigned boundary conditions with accounting of the change in internal stresses and the coordinates of crystallites, consequently involved into plastic deformation, ensure the formation of meso- and macro- bands of localized deformation. Shown in Fig. 2 is the result of the modelling of the process of plastic strain localization. It is seen that under the operation of the changing inhomogeneous stress field in the volume of polycrystal, the consequent development of the band structure of B type takes place, typical for the well-pronounced Portevin-Le Chatelier effect. A jump-like movement of the process of plastic strain localization along the working part of the specimen takes place in the form of the macrobands of localized shear.

![Figure 2: Sequential formation of meso- and macrobands of localized deformation.](image-url)
formed mesoband. A bunch of three mesobands forms the complete macroband (frame 5). At a definite distance from the macroband at the edge of the specimen, the zone of increasing tangential stresses takes place (dark background at the edge of the specimen in frame 8). The achievement of the critical value of defines the initiation at the edge of the specimen and development of the new macrobands of localized shear (frames 6, 9, 10). The process of nucleation and propagation of PLC-effect repeats periodically. After the process of deformation localization achieves an opposite end, the repeated formation of the macrobands, but in the conjugate direction of the maximum tangent stresses takes place (frame 12).

Shown in Fig. 3 is the dependence of the external stress on the numbers of grain involvement into plastic deformation. In fact this dependence is the loading diagram of modeled polycrystal, each n-act defines the definite quantum (2) of plastic deformation.

![Figure 3: Effect of intermittent flow on the curve of dependence of external stress](image)

Each peak is connected with the nucleation of the mesoband of localized plastic deformation at the edge of the specimen. The formation of the band occurs spontaneously under the decreasing external stress. The first mesoband among the three of them requires for its initiation the highest external stress in comparison with others. The lowest external applied stress matches the initiation of the second one.

In the course of deformation, the onset of the formation of a new macro-band occurs at a higher external applied stress in comparison with the previous one. Therefore, the change in the field of internal stress result in the effect of strain hardening.

The evolution of the development of plastic deformation in modeled polycrystal qualitatively repeat the regularities of PLC-band formation with well pronounced PLC-effect at the stage of jump-like propagation of PLC-band along the
working part of the specimen. As an example, shown in Fig. 4 is the sequence of PLC-formation in technical aluminum Al6061, identified by the displacement vectors field at the stage of the developed plastic deformation [4]. In the field of observation, from the beginning the first macroband appears (a), then the second one (2) and the third one (3). The jump of the process of localization from one position into another takes 15-18 seconds. The displacement vectors field in Fig. 4 is obtained when comparing of the metallographic frames of the sample surface, made in time interval 5 s. That is why displacement vectors register only the location of one band, which has formed during this time interval.

Figure 4: Jump-like propagation of the PLC band in technical aluminum Al6061 registered by displacement vectors field. $\varepsilon_p=8.3\text{-}8.5\%$ [11]

Conclusions

In the present paper, the modeling of the PLC-effect, which is accompanied by jump-like propagation of the macrobands of localized shear along the working part of the specimen have been performed by relaxation element method.

Given effect is shown to appear under the deformation of crystallites in the mode of pure shear. The macroband formation occurs by the mechanism of Lüder's band formation. The structure of the formed macroband consists of a number of mesobands, being oriented along the direction of maximum tangential stresses. The accumulation of the fields of internal stresses in the volume of polycrystal results in the effect of strain-hardening. The formation of each mesoband is accompanied by decreasing of external stress. The results of modelling qualitatively well agree with the known experimental data.

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References

