Transverse strength of continuous fiber metal matrix composites

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Abstract

The composite limit flow stress for transverse loading of metal matrix composites reinforced with continuous fibers is calculated using the finite element method. The focus of this work is to compare how different models for these composites influence the resulting composite flow behavior under transverse loading. Cell models with regular square and hexagonal arrangements [1] are compared with a so-called “embedded cell model” as developed by Dietrich et al. [2,3]. In the models for regularly arranged fibers, the hexagonal arrangement is seen to result in the highest limit stress for volume fractions less than about 0.42, whereas the square arrangement of fibers loaded in the direction of nearest neighbors provides greater strengthening at higher volume fractions. When the square arrangement is loaded 45° to the nearest neighbor direction, virtually no strengthening is seen. The interference of fibers with flow paths is seen to play an important role in the strengthening mechanism for these composites with regularly arranged fibers. Additionally, the influence of matrix hardening as a strengthening mechanism in these composites is seen to increase with volume fraction due to increasing fiber interaction. This is strengthening that results over and above the strengthening due to the addition of fibers. The embedded cell model is seen to agree quite well with the hexagonal model when the matrix has perfectly plastic behavior. However, when the matrix work hardens, the embedded cell model results in a higher asymptotic reference stress than calculated with the hexagonal model.

1. Introduction

Reinforcing lightweight metal matrices with ceramic fibers results in composites with excellent specific strength and creep resistance, for use in aerospace and transportation applications. See Ref. [1] for a more thorough listing of recent work on these composites. The use of continuous fiber reinforcements provides the greatest enhancement to the matrix, when loaded in the direction of the fiber axis. However, when loaded transverse to the direction of the fiber axis, the improvement in strength relative to the matrix material is limited. The transverse strength of these composites has been investigated for regular arrangements of fibers [1]. However, in most composites the fibers are arranged randomly. It is therefore of interest to examine models which attempt to capture this aspect of the composite. Böhm et al. [4] took unit cells and perturbed the position of the fiber in the cell and the boundary conditions on the cell surfaces to simulate ran-
dom arrangements. Nevertheless, these arrangements remain regular, although on a slightly larger scale. For all perturbations, the composite was seen to behave weaker than the unperturbed arrangement. Brockenbrough et al. [5] made calculations using a cell which reflected the fiber distribution taken from a transverse section of a micrograph of a real Al-B composite. When a large enough number of fibers is included in the cell, the composite response calculated becomes isotropic and does not change with an increase in cell size to include more fibers. They found the composite with randomly arranged fibers to be stronger than a composite with a regular hexagonal arrangement, which in turn was stronger than a composite with a square arrangement of fibers loaded diagonally. The strongest composite calculated was a square arrangement of fibers loaded in the direction of nearest neighbors.

The purpose of this paper is to further study the transverse behavior of continuous fiber composites with respect to the model used. In particular different cell models for regular fiber arrangements are compared with a so called "embedded cell model" for random fiber arrangements. Only circular fibers are investigated here. The elastic response of two phase composites is reasonably well understood [6], so the focus of this paper is limited to the fully developed plastic flow of these composites. The fibers are well bonded to the matrix so that no debonding or sliding is permitted at the interface. The finite element method is employed within the framework of continuum mechanics to carry out the calculations. Fig. 1 presents the regular fiber arrangements considered in this work. A square arrangement of fibers is shown in Fig. 1(a), with the loading directions at 0° and 45° indicated. Similarly, Fig. 1(b) represents a hexagonal arrangement of fibers, with the loading directions at 0° and 30° shown. Note that in the hexagonal arrangement the 0° direction is identical to the 60° direction and the 30° direction is identical to the 90° direction. Fig. 3 presents a schematic of the embedded cell model. In this model a single fiber is surrounded by a matrix layer, which is in turn surrounded by material which behaves with the composite response.

2. Model formulation

A plane strain model is used, since transverse strength is studied here and the fiber length is much greater than either the fiber diameter or fiber spacing. In fully plastic flow there is no plastic strain in the axial direction in a continuous fiber composite and therefore a plane strain model is appropriate.

The repeating cells used to model regular arrangements of fibers used for the calculations is shown in Fig. 2. The boundary conditions here are such that the lateral edges of the cell have zero average normal traction and zero shear traction. The loaded edges of the cell are given an average normal traction \( \bar{\sigma} \) and zero shear traction. The cell is forced to remain a rectangle and cannot rotate. The fiber is rigid. Modelling the fibers as rigid does not influence the fully plastic behavior of the composite although elastic and

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Fig. 1. (a) Square arrangement of fibers with primary loading directions, and (b) hexagonal arrangement of fibers with primary loading directions.
transient response prior to full plasticity will be influenced by the fiber rigidity. In addition, the fibers are well bonded to the matrix so that no motion is permitted to occur on the fiber perimeter.

The embedded cell model of Dietrich et al. [2,3] used in these calculations is shown in Fig. 3. In this model, a fiber is surrounded by matrix material, which is in turn surrounded by an equivalent composite material. The boundary conditions on this model are identical to those for the regularly arranged cells described above. However, in this case, the boundary between the matrix and the surrounding composite material is determined through compatibility. The dimensions of the model are such that the total width is \( L = 1 \), the radius of the matrix material is \( R = 0.2 \), and the radius of the fiber, \( r \), is related through the volume fraction \( f \), by \( f = r^2/R^2 \).

A continuum mechanics approach is used to model the composite behavior, thus eliminating the influence of size from the calculations. The uniaxial matrix stress-strain behavior is characterized by

\[
\sigma = \sigma_0 \left( \frac{\epsilon}{\epsilon_0} \right) = \epsilon E \quad \epsilon \leq \epsilon_0
\]

\[
\sigma = \sigma_0 \quad \epsilon > \epsilon_0 \tag{1a}
\]

for the perfectly plastic case, and by

\[
\sigma = \sigma_0 \left( \frac{\epsilon}{\epsilon_0} \right) = \epsilon E \quad \epsilon \leq \epsilon_0
\]

\[
\sigma = \sigma_0 \left( \frac{\epsilon}{\epsilon_0} \right)^N \quad \epsilon \geq \epsilon_0 \tag{1b}
\]

for the strain hardening cases. The parameter \( \sigma \) is the axial stress, \( \epsilon \) is the axial strain, \( \sigma_0 \) is the yield stress in tension, \( \epsilon_0 = \sigma_0/E \), \( E \) is Young's modulus, and \( N = 1/n \) is the strain hardening exponent.

With the embedded cell model, the equivalent composite material is initially assigned an arbitrary stress-strain response. Following the calculation this response is compared to the response of the cell composed of the embedded fiber-ma-
trix combination alone. If the responses are different the equivalent material is assigned the calculated response of the embedded cell, and the new total composite response is calculated. This process is repeated until a convergent solution is found. This typically requires 5 to 10 iterations.

The $J_2$ flow theory is employed with a von Mises yield criterion to characterize the rate-independent matrix material. The solutions were calculated incrementally, with the stress increment related to the strain increment through

$$
\dot{\sigma}_{ij} = \frac{E}{1 + \nu} \left( \dot{\epsilon}_{ij} + \frac{\nu}{1 - 2\nu} \dot{\epsilon}_{kk} \delta_{ij} \right) - \frac{3}{2\sigma^e} \frac{s_{ij}s_{kl}\dot{\epsilon}_{kl}}{\left( 1 + \frac{2}{3}(1 + \nu) \frac{E_i}{E - E_i} \right)},
$$

where $\dot{\sigma}_{ij}$ is the stress rate, $\dot{\epsilon}_{ij}$ is the strain rate, $s_{ij} = \sigma_{ij} - \sigma_{kk}/3$ is the deviatoric stress, $\sigma^e = \sqrt{\frac{2}{3}}s_{ij}s_{ij}$ is the tensile equivalent stress, $\nu$ is Poisson’s ratio, and $E_i$ is the current tangent modulus of the stress strain curve. The last term in Eq. (2) is zero for an elastic increment.

Fig. 4 presents the features of the overall stress-strain curves of primary concern in this work, following Zahl et al. [1]. For the case of fibers perfectly bonded to the matrix, the composite will necessarily harden with the same strain hardening exponent, $N$, as the matrix, when strains are in the regime of fully developed flow. At sufficiently large strains the composite behavior is then described by

$$
\bar{\sigma} = \bar{\sigma}_N \left( \frac{\bar{\epsilon}}{\epsilon_0} \right)^N,
$$

where $\bar{\sigma}$ is the overall stress, $\bar{\epsilon}$ is the overall strain, and $\bar{\sigma}_N$ is called the asymptotic reference stress. This is demonstrated in Fig. 4(a) for a non-hardening matrix, with $\bar{\sigma}_0$ being the limit flow stress. The asymptotic reference stress, $\bar{\sigma}_N$, can be determined by normalizing the composite stress by the stress in the matrix alone at the same overall strain, as indicated in figure 4b.

The ABAQUS finite element code [7] was employed using 8 noded 2-dimensional plane strain biquadrilateral elements. An IBM RS/6000 Model 540 work station was used to carry out the calculations, which typically took 30 minutes to compute a stress-strain curve for a specific model with one set of material parameters.

3. Results

3.1. Non-hardening matrices

Fig. 5 presents a plot of composite limit flow stress versus volume fraction for each of the regular arrangements considered here with a perfectly plastic matrix. At low strains, the hexagonal arrangement is seen to have the same limit flow stress or asymptotic reference stress whether loaded at $0^\circ$ or at $30^\circ$. This indicates a greater level of isotropy in the hexagonal arrangement over the square arrangement of fibers. The hexagonal arrangement provides slightly higher strengthening over the square arrangement at
volume fractions less than about 0.42, whereas the 0° loading of the square arrangement provides greater strengthening for volume fractions greater than this. The square arrangement loaded in the 45° direction results in no strengthening apart from that due to the plane strain condition, regardless of volume fraction. A marked rise in strengthening is seen in the square arrangement loaded at 0° at a volume fraction of about 0.4.

Similarly, the hexagonal arrangement experiences a marked rise in strengthening at a volume fraction of about 0.7. The improved strengthening of the hexagonal arrangement at low volume fractions is attributed to the restriction that shear deformations are only possible on 60° planes to bypass the fibers, rather than on the 45° planes on which the shear stress is maximum [1]. This effect is more marked with increasing volume fraction.

The sharp rises seen in these curves have been attributed to the shear bands being impinged by the fibers [1]. This effect is illustrated in Fig. 6. When a straight line can be drawn through the matrix in the maximum shear stress direction, there is no strengthening. Furthermore, the degree of strengthening correlates with the acuity of the plane just passing through the matrix tangential to the fibers. For the square arrangement with the 0° loading direction, the 45° shear planes become affected at a critical volume fraction of $f^* = 0.3927$, as shown in Fig. 6(a). Below this volume fraction, lines parallel to the maximum shear stress direction can pass entirely through the matrix. In this situation plastic flow can occur unimpeded by the fibers and so there is no strengthening. Above $f^* = 0.3927$ the plastic flow has to accommodate the presence of fibers and so constraint develops accompanied by composite strength. On the other hand, when the square arrangement is loaded in the 45° direction, the 45° shear planes are intact until the fibers contact one another at a volume fraction of $f_{\text{max}} = 0.7854$, Fig. 6(b). In the hexagonal arrangement, a 45° line representing the maximum shear stress direction will always pass through a fiber no matter what the volume fraction. Thus there will be a degree of constraint in the plastic flow of the matrix and therefore some composite strengthening at all volume fractions. This feature is apparent in Fig. 5. However, above a critical volume fraction of $f^* = 0.6802$, no straight line can be drawn entirely through the matrix. In this situation, the pattern of plastic flow must be more complex than that prevailing below $f^* = 0.6802$. This transition is accompanied by a significant volume fraction.

![Diagram](image)
increase in plastic constraint and consequently by more substantial composite strengthening, Fig. 5.

Fig. 7 presents the results for the hexagonal arrangement with the results for the embedded cell model when the matrix is perfectly plastic. It is seen here that the results for the embedded cell model in this case agrees extremely well with the results using the regular hexagonal cell model. This suggests that, when the matrix is perfectly plastic, the simpler hexagonal cell model can be used to approximate random arrangements of fibers, at a significant savings of computation time.

3.2. Hardening matrices

The asymptotic reference stress is plotted in Fig. 8 versus volume fraction for the regular hexagonal arrangement for strain hardening exponents of \( N = 0 \) (perfect plasticity), 0.1 and 0.2. Note here that the asymptotic reference stress in the hardening case is greater than the limit flow stress seen for the non-hardening matrices. This means that the composite gains additional strength over and above that associated with the constraining effects of the fibers. This is attributed to an increase in effective fiber diameter due to localized yielding and strain hardening of the matrix around the fibers. Initial yielding takes place adjacent to the fibers. This matrix material then hardens to some extent and thus has a higher yield stress than the remaining matrix material. As a result, even when the matrix is yielding everywhere, the deviatoric stress will be higher around the fibers than in the rest of the matrix. This effect increases with increasing strain hardening exponent, \( N \). The increase in strength due to fiber constraint during strain hardening increases with increasing volume fraction, but becomes significant only at volume fractions greater than the critical volume fraction, \( f^* \).

Stress-strain curves are plotted in Fig. 9 for the hexagonal arrangement, the square arrangement loaded in the direction of nearest neighbors, and for the embedded cell model for a strain hardening exponent \( N = 0.1 \) and for a volume fraction \( f = 0.5 \). The embedded cell model results in a stronger composite than the hexagonal model in this case, but not as strong as the square arrangement loaded in the direction of nearest neighbors. This indicates that, when the matrix strain hardens, the hexagonal cell model can no longer be used to accurately model composites with random fiber arrangements. However, the results for the embedded cell model are in qualitative agreement with the results of Brockenbrough et al. [5]. That is, the strength calculated using the embedded cell model lies between the strength calculated using the hexagonal cell model and that calculated using the square cell model loaded in
Fig. 9. Stress-strain curves for the square cell model (0°), the hexagonal arrangement and for the embedded cell model with a matrix work hardening exponent of $N = 0.1$. The volume fraction is $f = 0.5$.

the direction of nearest neighbors. The embedded cell model may thus be a useful tool in predicting the strength of composites with random fiber arrangements and hardening matrices, making use of a relatively simple geometry.

4. Closure

Calculations have been carried out with finite element cell models to investigate the effects of fiber arrangement and boundary conditions on the transverse stress-strain response of continuous fiber metal matrix composites. For regular fiber arrangements, the composite strength is seen to be sensitive to the fiber arrangement and loading direction, with a square arrangement of fibers loaded in the 0° direction being the strongest and a square arrangement loaded in the 45° direction being the weakest. Modest increments of strengthening in the hexagonal arrangement over the square arrangement for volume fractions below about 0.42 are noted. This is attributed to the 45° maximum shear planes being interrupted by fibers for all volume fractions in the hexagonal arrangement, but only at volume fractions higher than about 0.42 in the square arrangement. In the hexagonal arrangement 60° planes are present up to a volume fraction of about 0.68. Significant strengthening takes place when the volume fraction of fibers results in the impingement of available shear planes in the composite (45° shear planes for the square arrangement and 60° shear planes for the hexagonal arrangement). For the square arrangement loaded in the 45° direction, the 45° shear planes are present until the fibers contact one another. When the matrix behaves perfectly plastic, the regular hexagonal cell model gives essentially the same result as the embedded cell model for random fibers. Hardening can provide additional strengthening in these composites, even for the case of the square arrangement loaded at 45° [1]. When the matrix hardens, the hexagonal cell model no longer agrees with the embedded cell model. In this case the embedded cell model results in greater strengthening than the regular hexagonal arrangement, but not as much strengthening as provided by the regular square arrangement loaded at 0°. This indicates that the cell models for regular fiber arrangements cannot be used to model composites with random fiber arrangements when the matrix hardens.

References