Finite Element Method Simulation of Transverse Bridging in Fiber Reinforced Composites

Marc-Oliver NANDY, Nobuyuki TOHYAMA, Byung-Nam KIM, Manabu ENOKI, Siegfried SCHMAUDER* and Teruo KISHI

Research Center for Advanced Science and Technology, The University of Tokyo, 4-6-1, Komaba, Meguro-ku, Tokyo 153-8904
*Staatliche Materialprüfungsanstalt (MPA), University of Stuttgart, Pfaffenwaldring 32, 70569 Stuttgart, Germany

Delamination cracks in ceramic composite materials may be bridged by misaligned or inclined fibers at a shallow angle. The in situ observation of delamination cracks in a Si-Ti-C-O fiber-bonded ceramic composite material reveals that the bridging fibers are subjected to increasing tensile stresses as the crack opening displacement becomes larger. These stresses cause a crack closure pressure that is considered to contribute to steady state transverse fracture toughness. To relate the crack closure pressure to the material properties of fibers, matrix and their interface, a two-dimensional FEM model of a misaligned fiber bridging the crack wake at a shallow angle was constructed. Crucial mechanisms such as fiber debonding and frictional sliding along the debonded interface as well as matrix chipping were included. The crack closure pressure was simulated as a function of COD and the influences of these mechanisms were discussed. The toughening effect of bridging fibers was estimated and the obtained results were compared to the experimental data for a Si-Ti-C-O fiber-bonded ceramic composite material.

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1. Introduction

Continuous fiber-reinforced ceramic matrix composites are susceptible to delamination cracking due to the pronounced anisotropy in fracture resistance parallel and normal to the fiber orientation. Although delamination cracking is not necessarily followed by catastrophic failure, it has to be considered as an important damage mechanism because it reduces the structural stiffness. Moreover, the compressive strength as well as the shear strength of the material may decrease considerably. Thus, a better understanding of the mechanisms that govern delamination cracking is necessary to develop methods that contribute to suppress it.

It has been reported that the influence of reinforcing fibers on the transverse fracture toughness of ceramic matrix composites is an important factor for delamination crack growth. Some amount of misalignments which is intentionally introduced to uni-directional fiber-reinforced composites leads to an interaction between delamination cracks and fibers. Misaligned fibers or fiber bundles bridge the delamination crack at shallow angles and contribute to transverse fracture toughness which has the desirable effect of suppressing further growing of the crack. As the crack opening displacement increases, the bridging fibers start to debond from the matrix and peel off the crack faces, resulting in a limitation of the closure pressure and thus allowing greater displacement ranges than pin-jointed fibers would do.

The purpose of this paper is to construct a two-dimensional finite element method (FEM) model of a fiber bridging the crack wake at a shallow angle in order to simulate the correlation between closure pressure, matrix chipping and fiber debonding during delamination cracking. The model relates the crack closure pressure to basic constituent, fiber geometry and interface properties. Simulations were performed and discussed by using the experimental reference data for a Si-Ti-C-O fiber-bonded ceramic composite material. The results were also compared to a mechanical model which had been developed by Kaute et al. The present study contributes to the development of a procedure that allows the analysis of delamination cracking at elevated temperatures.

2. Material

The material used in this study was a unidirectional Si-Ti-C-O fiber-bonded ceramic composite (TyrannoHex; Ube Industries Co., Ltd., Japan) synthesized from pre-oxidized Si-Ti-C-O fibers by hot-pressing at 1750°C under a pressure of 40 MPa. The interstices between the fibers were packed with an oxide material which existed on the surface of the pre-oxidized Si-Ti-C-O fibers. The volume fraction of the oxide matrix is about 10 vol% and thus much lower than that of common ceramic-matrix fiber reinforced materials (30-70 vol%). Aligned turbostratic carbon was formed in situ as the result of a chemical reaction at the fiber-matrix interface during hot-pressing. This carbon layer plays an important role as regards the toughness properties of the material since it controls the sliding resistance of debonded fiber-matrix interfaces.

The material properties at room temperature reported for the Si-Ti-C-O fiber-bonded composite material are summarized in Table 1. The transverse fracture toughness was determined by performing Double-Torsion (DT)-tests with pre-notched specimens. The orientation of fibers was parallel to the direction of crack propagation. The fracture toughness can be determined by making use of the strain-energy release rate, \(G_{IC}\). The corresponding equation yields

\[
G_{IC} = \frac{1}{2} \frac{P_c^2}{t} \frac{dC}{da}
\]

where \(P_c\) is the critical load for crack propagation in a DT-specimen, \(t\) is the thickness of the specimen and \(dC/da\) is
Table 1. Constituent, Interface and Composite Properties of Si-Ti-C-O Fiber-Bonded Composite

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiber volume fraction, f&lt;sub&gt;v&lt;/sub&gt;</td>
<td>0.9</td>
</tr>
<tr>
<td>Matrix modulus, E&lt;sub&gt;matrix&lt;/sub&gt; (GPa)</td>
<td>95</td>
</tr>
<tr>
<td>Interface debonding energy, G&lt;sub&gt;c&lt;/sub&gt; (J/m&lt;sup&gt;2&lt;/sup&gt;)</td>
<td>0.2</td>
</tr>
<tr>
<td>Interfacial shear resistance, τ (MPa)</td>
<td>10-20</td>
</tr>
<tr>
<td>Fiber modulus, E&lt;sub&gt;f&lt;/sub&gt; (GPa)</td>
<td>180</td>
</tr>
<tr>
<td>Fiber diameter, d (μm)</td>
<td>9.45</td>
</tr>
<tr>
<td>Fiber strength, σ&lt;sub&gt;f&lt;/sub&gt; (MPa)</td>
<td>1.55</td>
</tr>
<tr>
<td>Gauge length, L&lt;sub&gt;gauge&lt;/sub&gt; (mm)</td>
<td>25</td>
</tr>
<tr>
<td>Weibull modulus, m</td>
<td>2.95</td>
</tr>
<tr>
<td>Composite tensile strength, σ&lt;sub&gt;c&lt;/sub&gt; (MPa)</td>
<td>400</td>
</tr>
<tr>
<td>Composite fracture toughness, K&lt;sub&gt;fc&lt;/sub&gt; (MPa√m)</td>
<td>17</td>
</tr>
<tr>
<td>Composite transverse fracture toughness, K&lt;sub&gt;tc&lt;/sub&gt; (MPa√m)</td>
<td>1 - 1.5</td>
</tr>
</tbody>
</table>

the relation between the compliance of the specimen, C, and the length of the initial notch, a, dC/da as well as K<sub>c</sub> had to be determined experimentally by using the compliance calibration method as described in Ref. 5. The average value after 7 tests was G<sub>c</sub> = 12.2 J/m<sup>2</sup>. Further experiments with double-cantilever beam (DCB) specimens revealed the values of approximately 12.5 J/m<sup>2</sup>. The fracture surfaces of the specimens were observed by a scanning electron microscope (SEM). Although the crack propagated mainly along the fiber-matrix interface, fibers were observed that bridged the crack in a shallow angle. The number of these fibers was estimated to be about 6/ mm<sup>2</sup>. The strength and the Weibull parameters for the Si-Ti-C-O fibers were determined by performing tensile tests with single fibers at a gauge length of 25 mm. Fiber diameters were measured by SEM for every single fiber.

In order to observe delamination cracking, in situ SEM four-point bending tests with pre-notched beam specimens were performed. After the load-displacement curve reached a maximum value, delamination cracking occurred. Figure 1 (a) shows a SEM micrograph of the zone near the notch with severe damage due to delamination cracks. Fibers that bridge a delamination crack at a shallow angle can be seen from Fig. 1 (b). The profile of these fibers originally follows the curved outline of a beam in bending, straightening out as the crack opening displacement increases. The cause for fiber bridging is fiber waviness.<sup>2</sup>

3. Modelling of fiber bridging

A two-dimensional FEM model of a fiber bridging the crack wake at a shallow angle during transverse fracture was constructed, as shown in Fig. 2. The mesh was refined in regions where high stress gradients are expected. In order to adjust the model to the real conditions, the geometry of in situ observed bridging fibers was taken into account. These observations suggest that the fibers are slightly curved where they are connected to the matrix. This curvature was taken into consideration as well as the geometrical dimensions of the Si-Ti-C-O fibers. The initial bridging angle of the modeled fiber has an angle of φ = 12°, the initial fiber length l<sub>f</sub> is 30 μm and the fiber diameter 10 μm. It should be noted that initial angles other than 12° may occur, too. But from in situ observations it can be seen that fibers with angles much larger than 12° hardly occur due to a sufficient alignment during processing, whereas misaligned fibers with very shallow angles do not bridge the crack. Thus, the chosen initial angle can be considered as the most frequent bridging angle.

At both ends, the fiber is embedded in the matrix (thickness: 5 μm), which consists of SiO<sub>2</sub> glass. Residual stresses due to the mismatch in the coefficient of thermal ex-
pansion and the fabrication procedure were neglected in this study. As indicated in Fig. 2, two mechanisms were assumed to occur when the crack opening displacement is increased: Debonding of the fiber from the interface together with sliding along the debonded interface and chipping of the matrix. Applying basic fracture mechanics, the debond length, \( x_d \), is calculated by the following equation:

\[
x_d = \frac{d}{4r} \left( \sigma - \sqrt{2} \frac{\sqrt{2}}{d} K_{Ic} \right)
\]

(2)

where \( \sigma \) is the tensile stress acting where the fiber enters the matrix, \( d \) is the fiber diameter, \( r \) is the interfacial shear resistance, and \( K_{Ic} \) is the mode II interface fracture toughness. Note that Eq. (2) which is used for a relatively thin matrix layer in the present study was originally derived for a single fiber embedded in an infinite matrix. In the present case of a thin matrix layer, the thickness as well as the mechanical boundary conditions of the thin matrix layer affect \( K_{Ic} \). The right term of Eq. (2) and thus \( K_{Ic} \) determines the onset of interfacial debonding. Therefore, the use of Eq. (2) in the present form may cause some uncertainties.

For simplicity, we assume that \( r \) is constant and that there is no difference between the debond shear stress and the frictional shear stress. Then, by applying the basic equation of frictional fiber sliding,\(^7\) the stress at the end of the debonded part of the fiber, \( \sigma(x_d) \), can then be calculated by

\[
\sigma(x_d) = \sigma - \frac{4 \pi x_d}{d}
\]

(3)

Because \( \sigma \) depends on the crack opening displacement, the debond length remains zero at the beginning of the bridging process. When the tensile stress in the fiber exceeds a certain threshold value (from Eq. (3) for \( x_d = 0 \)), debonding occurs and the debond length, \( x_d \), increases.

The in situ SEM observation of delamination cracks has shown that the matrix below the fiber starts to chip off when the bridging angle of the fiber attains a certain value (about 12°). Since the glass matrix of the Si-Ti-C-O fiber-bonded material is very brittle, its fracture was assumed to occur when the matrix tensile stress exceeds 100 MPa, a value reported for the tensile strength of SiO\(_2\) glass.

Within the FEM model, these two mechanisms were realized as follows: When debonding at the fiber-matrix interface occurred according to Eq. (2), the corresponding debond length was calculated and a crack with an opening much smaller than the fiber diameter and with a length equal to the calculated value of \( x_d \) was generated. The stress reduction of the fiber due to slippage relative to the fiber-matrix interface was then calculated by Eq. (3), with the average value for \( \sigma \) computed by FEM. At the location of the fiber-matrix interface where matrix chipping was expected to occur, the two surfaces of the crack were modeled in terms of a contact problem. When the crack opening displacement was increased, the fiber was distorted, loading the matrix similarly to a bar in bending. As soon as the mean tensile stress exceeded the tensile strength of the glass matrix at a certain node, all the outer elements of this node were removed, resulting in a longer bridging fiber length and a reduction of the bridging angle.

The model was loaded step by step by increasing the crack opening displacement. For each step, debonding and chipping were calculated and the geometry of the model was modified in the following step. At the beginning of the simulation, a small step width was chosen, but after several steps with increased crack opening displacement, larger step widths were set to reduce the time of calculation.

In order to calculate the crack closure pressure exerted by bridging fibers, two terms have to be considered: The force per fiber, \( f(u) \), and the number of bridging fibers per unit area, \( n(u) \). The force per fiber can be calculated from the average tensile fiber stress obtained by the FEM model. Considering that the fiber in the two-dimensional FEM model has a rectangular cross section, whereas the cross section of the actual fiber is circular, the force per fiber is determined as

\[
f(u) = \frac{2 \pi d^2}{4} \sigma(u) \sin \varphi
\]

(4)

where \( \sigma(u) \) is the average tensile fiber stress that evolves as the crack opening displacement \( u \) increases, and \( \varphi \) is the bridging angle as indicated in Fig. 1. The initial number of bridging fibers per unit area has to be estimated by SEM. It depends on the misalignment and volume fraction of fibers and diminishes by fiber failure. However, \( n(u) \) was considered to be constant in this study (\( n(u) = n_0 \)) for the following reason: The strength distribution of ceramic fibers is usually described by Weibull statistics. The strength quoted for the Si-Ti-C-O fibers in Table 1 is valid for the gauge length of 25 mm and the failure probability of 0.632. For a constant survival probability and fiber diameter, the strength is a function of fiber length:

\[
\frac{\sigma}{\sigma_{ref}} = \left( \frac{L_{ref}}{L} \right)^{1/m}
\]

(5)

where \( \sigma \) and \( \sigma_{ref} \) are defined as those stresses at which the failure probability of the fibers of length \( L \) and reference length \( L_{ref} = 0.632 \), respectively; \( m \) is the Weibull modulus. Inserting the values for \( \sigma_{ref}, m \) and \( L \) given in Table 1, a bridging Si-Ti-C-O fiber with the length of 0.5 mm would be able to sustain tensile stresses of about 6 GPa. Because the transverse fracture toughness is expected to be very low, it can be assumed that the fiber peels off the crack faces rather than failing by tensile stresses. SEM observations indicate that the length of bridging fibers ranges from 0.01 to 0.3 mm. With both \( f(u) \) and \( n(u) = n_0 \), the crack closure pressure yields the simple expression

\[
p(u) = f(u) n_0
\]

(6)

The closure pressure is used to determine the contribution of the bridging fibers to the steady state transverse fracture toughness \( G_{IIc} = G_{IIc} + \Delta G_{IIc} \) by using the following relation:

\[
\Delta G_{IIc} = \int_0^{u_m} p(u) \, du
\]

(7)

where \( u_m \) is the crack opening displacement at the end of the fully developed bridging zone remote from the crack tip and \( F \) is the mode I interface fracture resistance. Thus, Eq. (7) allows to relate the transverse fracture toughness properties to the crack closure pressure and thus to the material properties of fibers, matrix and their interface.

4. Results and discussion

Simulations were performed for interfacial shear strengths of 20 and 75 MPa, respectively. These values represent the lower half of the common range of \( r \) in fiber reinforced ceramic composites,\(^9\) whereas experimental results\(^10\) indicate that in the case of the Si-Ti-C-O fiber-bonded material used in this study \( r \) is lower than 20 MPa. Figure 3 shows the maximum tensile fiber stress, \( \sigma_{max} \), which occurs at the root of the fiber, and the average tensile fiber stress, \( \sigma \), which is measured at the center of the bridging length of the fiber, as a function of the crack opening displacement. At the beginning, when the crack opening displacement (COD) is very small, both the maximum tensile fiber stress as well as the average tensile fiber stress, increase more or less linearly with increasing COD. For com-
shear strengths of $r=20$ and $75 \text{ MPa}$. Initial debonding occurs at $\mu=\mu_i$, $\mu_i$ indicates the steady state, and at $\mu=\mu_f$ the fiber stress decreases due to pronounced fiber–matrix debonding at $\mu_f$ (see Fig. 4).

Comparison, the corresponding stress–COD curves of a curved bar derived from Beam-Column Theory are plotted in the same figure. The reason for the considerable difference between the FEM model and the calculated curves might be the fact that the matrix below the fiber was not considered in the latter. However, it can be seen that the slopes of the stress–COD curves reach similar values for both $\sigma_{\text{max}}$ and $\sigma$.

When debonding occurs, the stress–COD curve of the FEM model deviates from linearity. The critical COD for initial debonding, marked as $\mu_i$ in Fig. 3, was the same for both interfacial shear strengths. This can be explained by solving Eq. (2) for $x_3=0$ which yields the critical value of $\sigma$ at the fiber root where initial debonding occurs. This threshold value that causes debonding is only dependent on $K_{\text{ic}}$ and the fiber diameter, $d$, but not dependent on $r$. Therefore, the onset of debonding is the same for both $r=20$ and $75 \text{ MPa}$, respectively.

$K_{\text{ic}}$ is calculated from the mode II interface debond energy, $\Gamma$, which has been reported to exhibit very low values (0.1 to 0.2 $\text{ J/m}^2$ according to Ref. 9) in the case of ceramic matrix composites. In the present simulation, a constant value of $K_{\text{ic}}=0.19 \text{ MPa} \cdot \text{m}^{1/2}$ assumed which yields $\Gamma=0.2 \text{ J/m}$. It should be mentioned that $K_{\text{ic}}$ was estimated and that by varying $K_{\text{ic}}$ different values of $\mu_i$ would be obtained.

After initial debonding occurred, the fiber stress continues to rise with increasing COD, more pronounced for $r=75 \text{ MPa}$ than for $r=20 \text{ MPa}$. At a certain COD, as indicated approximately by $\mu_i$ in Fig. 3, the stress reaches a steady state, though oscillating around a mean value which is again higher for $r=75 \text{ MPa}$. Simulations were also performed up to larger COD values, however, a considerable change of the fiber stress was not observed. Figure 4 shows the total debond length as a function of the COD. It can be seen from this figure that the lower interfacial shear strength causes greater debond lengths. This is in agreement with the fact that the stress reduction by frictional sliding is lower for a lower interfacial shear strength. As can be expected, Fig. 4 indicates that the total debond length is given by a linear function of the COD. Progressive debonding causes the bridging length of the fiber to increase, resulting in stress relaxation as can be seen from Fig. 3 and Fig. 4: At the COD marked by $\mu_f$ in Fig. 4 the fiber–matrix debonding exhibits a pronounced leap, causing both $\sigma_{\text{max}}$ and $\sigma$ to decrease considerably, as marked by $\mu_f$ in Fig. 3. However, it should be noticed that debonding is usually a continuous process and thus the sizable variations of the debond length shown in Fig. 4 are a result of the stepwise calculation applied in this simulation.

On the other hand, the debond length affects the chipping length. The matrix starts spalling off because it can only withstand a certain amount of downward contact force exerted by the fiber. In this approach, the mean tensile stress was taken as the criterion for matrix fracture which might be an underestimation of the actual susceptibility of the matrix to this downward force. The evolution of matrix chipping is illustrated in Fig. 5 where the total chipping length is plotted as a function of the total debonding length. It can be seen from this figure that the evolution of matrix chipping follows a linear law with a similar slope for both the interfacial shear strengths of 20 and 75 MPa. However, the maximum of the total length of matrix that chipped away was larger for $r=20 \text{ MPa}$. The reason for this behavior is the longer debond length (Fig. 4) when the fiber debonds from the matrix, the downward force on the matrix increases due to a longer lever arm, since the part of the matrix which already debonded from the fiber is loaded similar to a bar in bending. Chipping is a discontinuous process which substantially contributes to the oscillation of the stress–COD curve (see Fig. 3). When a part of the matrix spalls off, the free length of the fiber increases, causing stress relaxation. Thus, debonding, interfacial sliding and matrix chipping are interacting with the stress–COD curves, as shown in Fig. 6, where the average debond stress, $\sigma_d$ (measured over the fiber diameter at the fiber root), and the average tensile fiber stress, $\sigma$ (measured at the center of the bridging length of the fiber) are plotted as a function of the COD for $r=75 \text{ MPa}$. Without taking frictional sliding into account, the $\sigma_d$ exhibits considerably higher values than $\sigma$. However, deduced by the frictional slipping component according to Eq.(3), $\sigma_d$ tends to oscillate around $\sigma$. 

Fig. 3. Maximum tensile fiber stresses and average tensile fiber stresses as functions of crack opening displacement for interfacial shear strengths of $r=20$ and $75 \text{ MPa}$. 

Fig. 4. Total debond lengths as functions of crack opening displacement for $r=20$ and $75 \text{ MPa}$. 

Fig. 5. Total chipping length as a function of the total debonding length for $r=20$ and $75 \text{ MPa}$. 
The results obtained by the FEM simulation have been compared to the bridging model developed by Kaute et al. The model employs both simple mechanics and Weibull statistics to express the crack closure pressure as a function of the material properties of fibers, matrix and their interface. Figure 7 shows the closure pressure as a function of COD for both the FEM calculation performed in this study and the model by Kaute et al. for $\tau = 20$ MPa. The closure pressure was calculated by using Eqs. (4) and (6), and the initial number of fibers bridging the crack wake was set $n_0 = 6 \text{ mm}^{-2}$ in Eqs. (A1) and (A2). For a mode I fracture toughness of $K_{Ic} = 0.7 \text{ MPa}\sqrt{\text{m}}$, the agreement between the models would exhibit a similar characteristic. At a crack opening displacement of 0.05 mm, for example, the contribution of the bridging fibers to the transverse fracture toughness of the Si-Ti-C-O fiber-bonded composite material is predicted to be $\Delta G_{F} = 1 \text{ J/m}^2$. Comparing this value to the experimentally obtained transverse fracture toughness of the material ($G_{F} = P_{c} + \Delta G_{F}$), exhibiting values between 12.2 and 12.5 J/m$^2$ it appears that the predictions of these models are quite reasonable. Since the number of bridging fibers is small in the Si-Ti-C-O fiber-bonded composite material, the contribution of the bridging fibers to the transverse fracture toughness remains on a relatively low level. Fiber failure does not seem to be significant as long as the crack opening displacement remains small. However, for larger displacements, fiber failure may occur, reducing the closure pressure and leading to an upper limit of the rising R-curve. Since such large COD-values are definitely unacceptable for any kinds of technical applications, it seems to be sufficient to consider only the initial rising part of the R-curve where fiber failure can be neglected. In this range, the FEM model allows reasonable predictions of the transverse fracture toughness of fiber reinforced composites.

5. Conclusions

(1) Delamination cracks in fiber reinforced ceramic composites are found to be bridged by fibers or fiber bundles in a shallow angle. These bridging fibers or fiber bundles have been shown to contribute to the transverse fracture toughness by exerting a closure pressure on the crack faces.

(2) A model for a single bridging fiber was applied to the Finite Element Method taking into account the geometrical properties of a curved fiber. The material properties of a Si-Ti-C-O fiber-bonded ceramic composite material were used for the model calculations.

(3) The crack closure pressure as a function of the crack opening displacement was simulated. It was found that the debonding and matrix chipping processes lead to stress-COD curves that differ significantly from those calculated by the beam-column theory. The crack closure pressure, debond length and the amount of matrix chipping are depending on the interfacial shear strength of the composite. The predicted contribution to the transverse fracture toughness of the Si-Ti-C-O fiber-bonded ceramic composite material in the FEM model yielded satisfactory results.

Appendix

According to Ref. 2, the closure pressure induced by bridging fibers along the crack faces is described by the following two equations; the first one is valid for the early stage of the bridging pro-

![Fig. 6. Average debond stress with and without frictional slipping as functions of crack opening displacement for $\tau = 75$ MPa. The average fiber stress is also plotted for comparison.](image)

![Fig. 7. Crack closure pressures plotted as functions of crack opening displacement. The result obtained by the FEM simulation for $\tau = 20$ MPa are compared to the analytical model of Kaute et al. The arrows mark the onset of debonding.](image)

![Fig. 8. Predicted contribution of fiber bridging to the transverse fracture toughness of the Si-Ti-C-O fiber-bonded composite material.](image)
cess before the crack closure pressure, \( p(u) \), reaches its maximum:

\[
p(u) = \frac{3\pi E_d}{64 + \frac{E_d}{E_b}} \left( \frac{u}{l_0} \right) + \frac{\pi d^2}{4} \left( \frac{u}{l_0} \right) \left( \frac{u}{l_0} \right)^2
\]

(A1)

Due to matrix chipping, the maximal fiber force is limited to a certain value. As the matrix peel-off process exposes the fiber more and more to the maximum fiber stress, the number of fibers decreases via fiber failure. At this state, the closure pressure is expressed by

\[
p(u) = \frac{C_d u}{\sin \varphi_{\text{max}, \text{ref}}} \left( \frac{\sigma_b}{\sigma_{\text{ref}}} \right)^n \exp \left( u - u^* \right)
\]

(A2)

where \( u^* \) is a mean crack length parallel to the fiber over which the steady state force acts, \( \sigma_b \) stands for the mean maximum bending stress at the root of the fiber, \( C_d \) is a dimensionless correction factor smaller than unity to allow the comparison between the mean bending stress and the reference tensile stress, \( \varphi_{\text{max}} \) is the steady state bridging angle and \( u^* \) is the reference crack opening up to which the bridging fibers remain intact. In the present study \( u^* \) was 0.002 mm, \( C_d = 0.7 \), and \( \sigma_b = 5.42 \) GPa. In situ observations indicate \( \varphi_{\text{max}} \) is about 12° and \( u^* \) about 0.0135 mm. All other parameters are given in Table 1.

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